(a) Pipeline scour by currents. In steady currents the equilibrium scour depth beneath a pipeline is thought to be a function of pipe diameter, pipe roughness, pipe Reynolds number, and Shields parameter. For clear water scour, when mean flow velocity, U, is less than the critical velocity, U_c, maximum scour depth can be calculated using the following equation from Hoffmans and Verheij (1997)

$$\frac{S_m}{D} = \frac{\mu}{2}\left(\frac{U}{U_c}\right) \tag{VI-5-270}$$

where

$$\mu = \left(\frac{k_s}{12\,D}\right)\ln\left(\frac{6\,D}{k_s}\right) \tag{VI-5-271}$$

and

D = pipe diameter

h = water depth

U = depth averaged flow

U_c = critical depth-averaged flow velocity

k_s = effective bed roughness, $k_s = 3\,d_{90}$ (k_s must have the same units as D)

When $U/U_c > 1$, live-bed scour occurs, and in this case Sumer and Fredsøe (1992) stated that pipe Reynolds number only influences flow around smooth pipes and the influence of Shields parameter is minor. They recommended the simple equation for predicting maximum equilibrium scour depth. The 0.1-value represents the standard deviation of the data, so a conservative estimate of scour would be S_m/D=0.7.

$$\frac{S_m}{D} = 0.6 \pm 0.1 \tag{VI-5-272}$$

(b) Pipeline scour by waves. Oscillatory bottom velocities under waves create piping conditions beneath pipelines in the same manner as steady currents. Sumer and Fredsøe (1991) gave a criterion for onset of scour under waves based on a small number of laboratory experiments. This criterion is

$$\frac{e_{cr}}{D} = 0.1\ln(KC) \tag{VI-5-273}$$

where e_{cr} is the critical embedment (depth of pipeline burial beyond which no scour occurs), and KC is the Keulegan-Carpenter number, given by Equation VI-5-256 with D as the denominator. Scour is unlikely to occur for values of $e_{cr}/D > 0.5$ (half buried pipe). Sumer and Fredsøe (1990) studied scour beneath a bottom-resting pipeline under wave action. Their laboratory data, combined with that of an earlier researcher, indicated that live-bed scour was strongly related to Keulegan-Carpenter number and pipe diameter, while only weakly influenced by Shields parameter and pipe roughness. The data were well represented over a wide range of Keulegan-Carpenter number ($2 < KC < 300$) by the empirical expression

$$\frac{S_m}{D} = 0.1\sqrt{KC} \tag{VI-5-274}$$

Klomb and Tonda (1995) presented a modified version of Equation VI-5-274 that included allowance for partial embedment, *e*, of the pipeline, i.e.,

$$\frac{S_m}{D} = 0.1\sqrt{KC}\left(1 - 1.4\frac{e}{D}\right) + \frac{e}{D} \tag{VI-5-275}$$

with scour depth taken relative to the undisturbed bed. Equation VI-5-275 is valid for values of $e/D < 0.5$ (Hoffmans and Verheij 1997).

(c) Pipeline scour by waves and currents. Sumer and Fredsøe (1996) conducted laboratory tests of pipeline scour due to combined waves and currents covering a range of *KC* from 5 to about 50 with codirectional currents. The general trend, regardless of the value of *KC*, was for scour depth to initially decrease as current is increased from zero. At higher values of current, maximum scour depth approaches the value given by Equation VI-5-272 for currents alone. Sumer and Fredsøe (1996) provided empirical design equations based on the laboratory experiments; but for values of *KC* between 40 and 50 maximum scour depth is almost the same as the estimate for currents alone.

(d) Pipelines in the nearshore. Pipelines traversing the surfzone may be damaged if exposed to breaking waves and strong longshore currents. Little design guidance is available other than the fact that additional scour will occur once the pipeline is exposed. The burial depth for a pipeline through the nearshore should exceed in all places the expected bottom profile lowering that might occur over the life of the pipeline. This can be estimated using profile-change models or from long-term beach profile data.

(5) Other scour problems. Some coastal projects may include structural elements or hydrodynamic flow conditions that are typically associated with inland waterways or estuaries. Structures such as storm surge barriers, discharge control structures, or large pad footings may experience scour around their foundations due to currents or combined waves and currents.

(a) Hoffmans and Verheij (1997) provided a summary of techniques for estimating maximum scour for a number of situations that may be applicable to coastal projects:

- Scour downstream of sills and stone blankets due to currents.
- Scour downstream of hard bottoms due to horizontal submerged jets.
- Scour at control structures due to plunging jets.
- Scour at two- and three-dimensional culverts.
- Scour at abutments and spur dikes.

(b) See Hoffmans and Verheij (1997) for further details and associated technical literature.

c. Design of scour protection. Toe protection in the form of an apron is needed to prevent toe scour which may destabilize or otherwise decrease the functionality of a coastal structure. The apron must remain intact under wave and current forces, and it should be flexible enough to conform to an initially uneven sea floor. Scour apron width and required stone size for stability are related to wave and current intensity, bottom

material, and structure characteristics such as slope, porosity, and roughness. Design guidance for scour protection is based largely on past successful field experience combined with results from small-scale laboratory tests. Special attention is needed where scour potential is enhanced such as at structure heads/ends, at transitions in structure composition, or at changes in structure alignment. This section provides general design guidance for scour aprons; however, this guidance should be considered preliminary. Projects requiring absolutely stable scour blankets should have proposed designs tested in a physical model. Hales (1980) surveyed scour protection practices in the United States and found that the minimum scour protection was typically an extension of the structure bedding layer and any filter layers. The following minimum rules-of-thumb resulted from this survey: minimum toe apron thickness - 0.6 m to 1.0 m (1.0 m to 1.5 m in northwest U.S.); minimum toe apron width - 1.5 m (3 m to 7.5 m in northwest U.S.); material - quarrystone to 0.3 m diameter, gabions, mats, etc. These rules-of-thumb are inadequate when the water depth at the toe is less than two times the maximum nonbreaking wave height at the structure or when the structure reflection coefficient is greater than 0.25 (structures with slopes greater than about 1:3). Under these more severe conditions use the scour protection methods summarized in the following sections for specific types of coastal structures.

(1) Scour protection for vertical walls.

(a) Vertical-front structures consist of large caisson-type gravity structures, gravity retaining walls, and cantilevered or anchored sheet-pile retaining walls. Toe protection design for larger vertical-front gravity structures subjected to waves is covered in Part VI-5-3*d*, "Toe stability and protection."

(b) For cantilevered or anchored retaining walls, Eckert (1983) proposed toe protection in the form of a scour apron constructed of quarrystone. The main purpose of the apron is to retain soil at the toe and/or to provide sufficient weight to prevent slip failure (see Figures VI-2-69 and VI-2-70). From geotechnical considerations the width (W) of the scour apron should be approximately

$$W = \frac{d_e}{\tan(45^o - \phi/2)} \approx 2.0\, d_e \qquad \text{(VI-5-276)}$$

where d_e is the depth of sheet-pile penetration below the seabed, and φ is the angle of internal friction of the soil (varies from about 26 deg to 36 deg). The width of the scour apron based on hydrodynamic criteria was given by Eckert as the greater of

$$W = 2.0\, H_i \qquad or \qquad W = 0.4\, d_s \qquad \text{(VI-5-277)}$$

where H_i is the incident wave height and d_s is the depth at the structure toe. Selected scour apron design width will be the greater of Equations VI-5-276 and VI-5-277.

(c) Eckert (1983) noted that gravity retaining walls do not require the apron to be as wide as needed for cantilevered walls. In this case, he recommended that scour apron width be about the same as the nonbreaking incident wave height.

(d) Determining the toe apron quarrystone size depends on the hydrodynamic conditions. They are as follows:

- Waves. If retaining walls are exposed to vigorous wave conditions, the toe quarrystone should be sized using the guidance given by Figure VI-5-45 (Part VI-5-3*d* "Toe stability and protection," and the apron thickness should be equal to either two quarrystone diameters or the minimum given in the prior rules-of-thumb, whichever is greater.

- Currents. If strong currents flow adjacent to the wall, toe quarrystone should be sized using the guidance provided in Part VI-5-3*f*, "Blanket stability in current fields."

- Waves and Currents. If both waves and strong currents impact the toe adjacent to a vertical retaining wall, estimate the size of the apron quarrystone for the waves alone and for the current alone. Then increase whichever is larger by a factor of 1.5 (Eckert 1983).

(e) In Sumer and Fredsøe's (1996) study of scour around the head of a vertical breakwater, laboratory tests were conducted to establish a relationship for the width of a scour apron that provides adequate protection against scour caused by wave-generated lee-wake vortices. Their empirical formula was given as

$$\frac{W}{B} = 1.75\,(KC - 1)^{1/2} \qquad \text{(VI-5-278)}$$

where B is the diameter of the vertical breakwater circular head and KC is the Keulegan-Carpenter number given by Equation VI-5-256. Sumer and Fredsøe cautioned that this estimation of apron width may be inadequate in the presence of a current or for head shapes other than circular. Scour apron stone sizes are determined using the methods outlined in Part VI-5-3*d*, "Toe stability and protection."

(2) Scour protection for sloping structures.

(a) Scour protection for sloping structures exposed to waves is typically provided by the toe protection. Part VI-5-3*d*, "Toe stability and protection," presents guidance on the design of toe protection. Additional scour protection is sometimes needed at sloping-front structures to prevent scour by laterally-flowing currents. Strong tidally-driven currents adjacent to navigation jetties can scour deep trenches that may destabilize the jetty toe and result in slumping of the armor layer. Because prediction of the location and extent of potential scour is not well advanced, scour blankets are often not installed until after realization that scour has occurred. Depending on the scour hole configuration, it may be necessary to backfill the scour hole before placing a scour blanket, and the necessary extent of the protection is determined in part by the extent of the existing scour, by past experience, and by the judgment of the engineer. An understanding of the flow regime will help assure that the scour problem will not reoccur downstream of the scour protection blanket. Stone size for scour protection from currents is given in Part VI-5-3*f*, "Blanket stability in current fields." Bass and Fulford (1992) described the design and installation of scour protection along the south jetty of Ocean City Inlet in Maryland.

(b) Fredsøe and Sumer's (1997) laboratory study of wave-induced scour at the rounded heads of rubble-mound structures included design suggestions for scour protection. The width of the scour apron from the structure toe to outer edge was given by

$$\frac{W}{B} = A_1\,(KC) \qquad \text{(VI-5-279)}$$

where B is the breakwater head diameter at the bed and KC is given by Equation VI-5-256. Complete scour protection is provided with A_1 = 1.5 whereas a value of A_1 = 1.1 will result in relatively minor scour at the outer edge with a depth equal to about 0.01 B. Scour apron stone size are determined using the methods outlined in Part VI-5-3*d*, "Toe stability and protection."

(3) Scour protection for piles.

(a) Vertical piles and piers exposed only to currents can be protected against scour by placement of scour aprons constructed of stone or riprap, gabions, concrete mattresses, or grout-filled bags. Riprap aprons should be designed according to the relationships given in Part VI-5-3*f*, "Blanket stability in current fields." Options other than riprap or stone should be tested in physical models.

(b) Based on an earlier report by Bonasoundas (in German), Hoffmans and Verheij (1997) recommended that minimum width for the horizontal extent of the scour apron around circular piers be specified as a function of pile diameter, *B*. Upstream of the pile, and to both sides, apron width is 2.5 *B*. Downstream the apron elongates to a width of 4.0 *B* as illustrated on Figure VI-5-123. Elongation in both directions is necessary for alternating tidal currents.

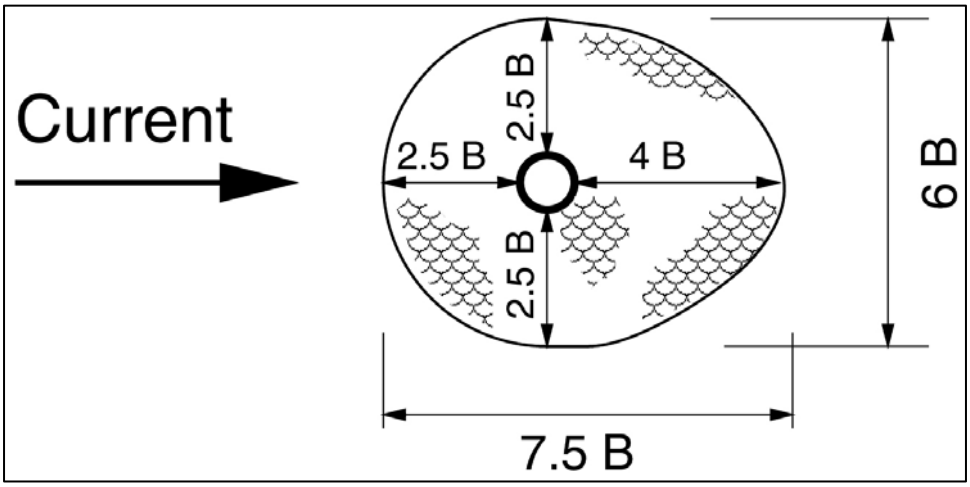

Figure VI-5-123. Scour apron for vertical pile in a current

(c) An alternative recommendation was given by Carstens (1976) who found that scour apron width was a function of maximum scour depth (S_m) at the pile, i.e.,

$$\frac{W}{S_m} = \frac{F_s}{\tan \phi} \tag{VI-5-280}$$

where φ is the bed material angle of repose and F_s is a factor of safety.

(d) General recommendations for specifying apron width for different shaped piers and pilings, or for groups of piles, are lacking. In these cases laboratory model tests are needed to assure adequate scour protection. Past experience on other successful projects or case histories reported in the literature can also serve as design guidance (e.g., Edge et al. 1990; Anglin et al. 1996).

(e) Similar protective measures can be deployed to prevent scour around piles by wave action. However, guidance is also lacking on how to design stable scour aprons in wave environments (Sumer and Fredsøe 1998a), and the best recourse is site-specific model tests. As a rule-of-thumb, the horizontal extent of the apron should be approximately twice the predicted scour depth.

(4) Scour protection for submerged pipelines.

(a) Submerged pipelines can be protected by either burying the pipeline in a trench or by covering the pipeline with a stone blanket or protective mattress. Protected pipelines are less susceptible to trawler damage

and less likely to suffer damage caused by differential scour that leaves portions of the pipeline suspended between support points.

(b) Outside the active surfzone, burial depth is a function of local wave and current climate, sediment properties, and liquefaction potential. Usually the excavated material can be used as backfill provided it is sufficiently coarse to avoid buildup of excessive pore pressures which could lead to liquefaction and vertical displacement of the pipeline (Sumer and Fredsøe 1998a). Pipelines traversing the surfzone should be buried at an elevation lower than the anticipated erosion that would occur over the projected service life of the pipeline. Generally, stone blankets or mattresses are not considered effective protection in the surfzone because the elements must be designed to withstand the intense action of breaking waves.

(c) Pipelines resting on the bottom can be protected from being undermined by stabilizing the adjacent bed with a stone blanket having a horizontal width less than the extent of expected scour. Hjorth (1975) reported that covering at least the bottom half of the pipeline, as shown in the upper part of Figure VI-5-124, provides sufficient protection as evidenced by field experience. The alternative is to cover the pipeline completely with a stone blanket consisting of two or more filter layers as illustrated by the lower sketch of Figure VI-5-124. Stability of the uppermost stone layer requires that the shields parameter (Equation III-6-43) based on stone diameter must be less than the critical value for incipient motion. Stone blanket placement can be accomplished by dumping stone from the surface, provided the falling stones are not so large as to damage the pipeline on impact.

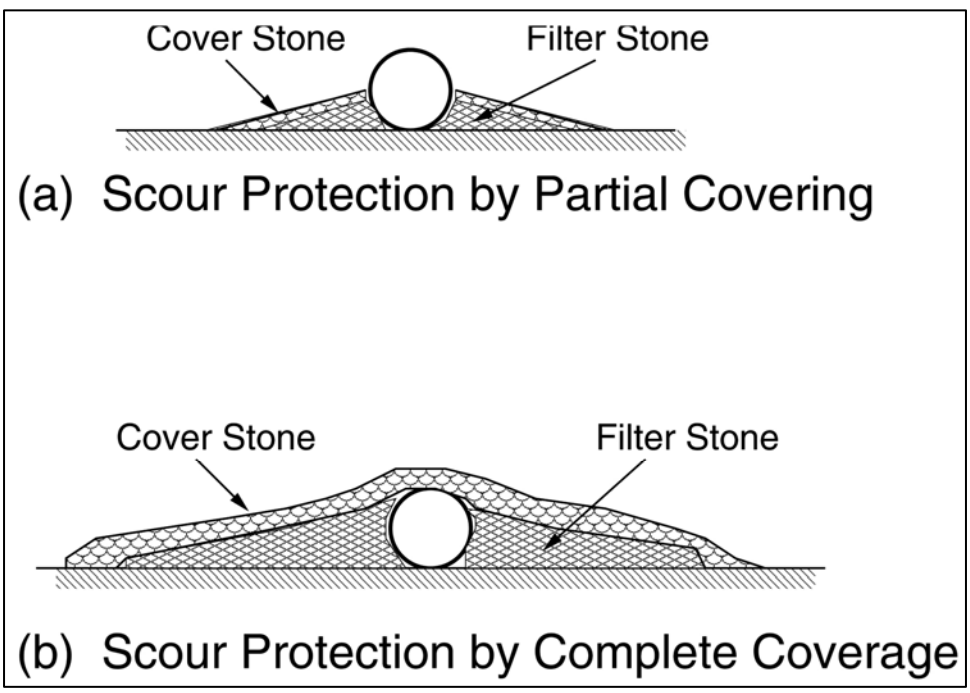

Figure VI-5-124. Stone blanket scour protection for submerged pipelines

(d) Various types of scour mattresses have also been used effectively to protect pipelines. Mattresses may be economical when stone is not readily available; however, special mattress placing equipment is usually required. Hoffmans and Verheij (1997) illustrated several types of mattresses.

VI-5-7. Wave Forces on Slender Cylindrical Piles

a. Introduction.

(1) Frequent use of pile-supported coastal and offshore structures makes the interaction of waves and piles of significant practical importance. The basic problem is to predict forces on a pile due to the wave-associated flow field. Because wave-induced flows are complex, even in the absence of structures, solution of the complex problem of wave forces on piles relies on empirical coefficients to augment theoretical formulations of the problem. This section is meant to be only an introduction to estimating forces and moments on slender cylindrical piles. For more detailed analysis see the literature related to ocean engineering and the design of offshore facilities.

(2) Variables important in determining forces on circular piles subjected to wave action are shown in Figure VI-5-125. Variables describing nonbreaking, monochromatic waves are the wave height H, water depth d, and either wave period T, or wavelength L. Water particle velocities and accelerations in wave-induced flows directly cause the forces. For vertical piles the horizontal fluid velocity u and acceleration du/dt and their variation with distance below the free surface are important. The pile diameter D and a dimension describing pile roughness elements k are important variables describing the pile. In this discussion the effect of the pile on the wave-induced flow is assumed negligible. Intuitively, this assumption implies that the pile diameter D must be small with respect to the wavelength L. Significant fluid properties include the fluid density ρ and the kinematic viscosity v. In dimensionless terms, the important variables can be expressed as follows:

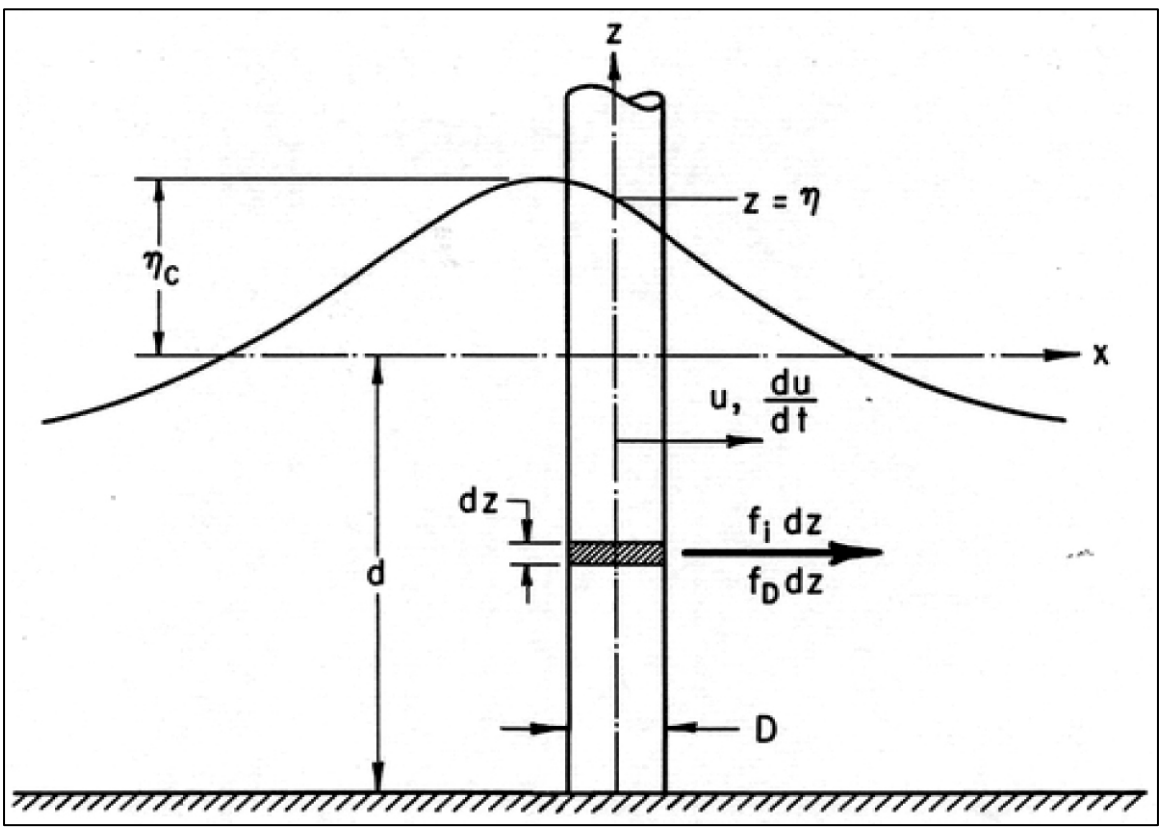

Figure VI-5-125. Definition sketch of wave forces on a vertical cylinder

$$\frac{H}{gT^2} = \text{dimensionless wave steepness}$$

$$\frac{d}{gT^2} = \text{dimensionless water depth}$$

$$\frac{D}{L} = \text{ratio of pile diameter to wavelength (assumed small)}$$

$$\frac{k}{D} = \text{relative pile roughness}$$

$$\frac{HD}{T\nu} = \text{a form of the Reynolds number}$$

(3) Given the orientation of a pile in the flow field, the total wave force acting on the pile can be expressed as a function of these dimensionless parameters. The variation of force over the length of the pile depends on the mechanism by which the water particle velocities and accelerations cause the forces. The following analysis relates the local forces acting on a section of pile element of length *dz* to the local fluid velocity and acceleration that would exist at the center of the pile if the pile were not present. Two dimensionless force coefficients, an inertia (or mass) coefficient C_M and a drag coefficient C_D, are used to establish the wave-force relationships. These coefficients are determined by experimental measurements of force, velocity, and acceleration or by measurement of force and water surface profiles, with accelerations and velocities inferred by assuming an appropriate wave theory.

(4) In the following section it is initially assumed that the force coefficients C_M and C_D are known to illustrate calculation of forces on vertical cylindrical piles subjected to monochromatic waves. Selection of C_M and C_D follows in Part VI-5-7*c*. Experimental data are available primarily for the interaction of nonbreaking waves and vertical cylindrical piles; and consequently, specific design guidance can be given for this common situation.

b. Vertical cylindrical piles and nonbreaking waves.

(1) Basic concepts. Morison et al. (1950) suggested that the horizontal force per unit length of a vertical cylindrical pile subjected to waves is analogous to the mechanism by which fluid forces on bodies occur in unidirectional flow, and this force can be expressed by the formulation

$$f = f_i + f_D = C_M \rho \frac{\pi D^2}{4} \frac{du}{dt} + C_D \frac{1}{2} \rho D u \,|u| \qquad \text{(VI-5-281)}$$

where

f_i = inertial force per unit length of pile

f_D = drag force per unit length of pile

ρ = mass density of fluid

D = pile diameter

u = horizontal water particle velocity at the axis of the pile (calculated as if the pile were absent) total

$\frac{du}{dt}$ = horizontal water particle acceleration at the axis of the pile (calculated as if the pile were absent)

C_D = drag hydrodynamic force coefficient

C_M = inertia or mass hydrodynamic force coefficient

(a) The inertia force term f_i is of the form obtained from an analysis of the force on a body in an accelerated flow of an ideal nonviscous fluid. The drag force term f_D is the drag force exerted on a cylinder in a steady flow of a real viscous fluid. The drag force f_D is proportional to u^2 and acts in the direction of the velocity u. To retain the correct direction sign, u^2 is written as $u\,|u\,|$. Although these remarks support the soundness of the formulation of the problem as given by Equation VI-5-281, it should be emphasized that expressing total force by the terms f_i and f_D is an assumption justified only if it leads to sufficiently accurate predictions of wave force as evidenced by ample measurements.

(b) Because the quantities u and du/dt in Equation VI-5-281 are defined as the values of these parameters at the axis of the pile, it is apparent that the influence of the pile on the flow field a short distance away from the pile has been neglected. Using linear wave theory MacCamy and Fuchs (1954) analyzed theoretically the problem of waves passing a circular cylinder. Their analysis assumed an ideal nonviscuous fluid and led to an inertia force having the form given for f_i under special conditions. Although their theoretical result is valid for all ratios of pile diameter to wavelength, D/L, the inertia force was found to be nearly proportional to the acceleration du/dt for small values of D/L (where L is wavelength calculated by linear theory). This theoretical result provides an indication of how small the pile should be for Equation VI-5-281 to apply, and the restriction is given as

$$\frac{D}{L} < 0.05 \qquad \text{(VI-5-282)}$$

where L is calculated by linear wave theory. This restriction will seldom be violated for slender pile force calculations; however, the restriction may be important when applying Equation VI-5-281 to larger structures such as cylindrical caissons.

(c) To apply Equation VI-5-281 it is necessary to choose an appropriate wave theory for estimating u and du/dt from values of wave height H, wave period T, and water depth d; and for that particular wave condition appropriate values of C_D and C_M must be selected.

(2) Calculation of forces and moments. For structural design of a single vertical pile, it is often unnecessary to know in detail the distribution of forces over the height of the pile. Instead, the designer needs to know the total maximum force and the total maximum moment about the mud line ($z = -d$) acting on the pile. The total time-varying force and the time-varying moment acting about the mud line is found by integrating Equation VI-5-281 between the bottom and the free surface, i.e.,

$$F = \int_{-d}^{\eta} f_i\, dz + \int_{-d}^{\eta} f_D\, dz = F_i + F_D \qquad \text{(VI-5-283)}$$

$$M = \int_{-d}^{\eta} (z + d)\, f_i\, dz + \int_{-d}^{\eta} (z + d)\, f_D\, dz = M_i + M_D \qquad \text{(VI-5-284)}$$

In general form these quantities may be written

$$F_i = C_M \rho g \frac{\pi D^2}{4} H K_i \tag{VI-5-285}$$

$$F_D = C_D \frac{1}{2} \rho g D H^2 K_D \tag{VI-5-286}$$

$$M_i = C_M \rho g \frac{\pi D^2}{4} H K_i d S_i = F_i d S_i \tag{VI-5-287}$$

$$M_D = C_D \frac{1}{2} \rho g D H^2 K_D d S_D = F_D d S_D \tag{VI-5-288}$$

in which C_D and C_M have been assumed constant, and where K_i, K_D, S_i, and S_D are dimensionless parameters that depend on the specific wave theory used in the integrations. In the following sections values of the inertia coefficient C_M and drag coefficient C_D are assumed to be known constants. (Part VI-5-7*c* covers estimation of C_M and C_D.)

(a) Linear wave theory. The force on a slender cylindrical pile can be estimated using linear wave theory, but the result is limited to situations where linear wave theory provides a reasonable approximation of the wave kinematics. This implies small amplitude waves and greater depths. Also recall that any wave force on the pile above the swl will not be included in the estimate. Nevertheless, it is instructive to examine Equation VI-5-281 when linear wave theory is applied.

- With the pile center line located at $x = 0$, as shown in Figure VI-5-125, the equations from Part II-1, "Water wave mechanics" for surface elevation (Equation II-1-19), horizontal component of local fluid velocity (Equation II-1-22), and horizontal component of local fluid acceleration (Equation II-1-24) are respectively

$$\eta = \frac{H}{2} \cos\left(\frac{2\pi t}{T}\right) \tag{VI-5-289}$$

$$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh[2\pi d/L]} \cos\left(\frac{2\pi t}{T}\right) \tag{VI-5-290}$$

$$\frac{du}{dt} \approx \frac{\partial u}{\partial t} \approx \frac{g\pi H}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh[2\pi d/L]} \sin\left(-\frac{2pit}{T}\right) \tag{VI-5-291}$$

- Introducing Equations VI-5-290 and VI-5-291 for u and du/dt into Equation VI-5-281 gives the following expressions for the inertia force and drag force.

$$f_i = C_M \rho g \frac{\pi D^2}{4} H \left[\frac{\pi}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh[2\pi d/L]}\right] \sin\left(-\frac{2pit}{T}\right) \tag{VI-5-292}$$

$$f_D = C_D \frac{1}{2} \rho g D H^2 \left[\frac{gT^2}{4L^2} \left(\frac{\cosh[2\pi(z+d)/L]}{\cosh[2pid/L]} \right)^2 \right] \cos\left(\frac{2\pi t}{T}\right) \left| \cos\left(\frac{2\pi t}{T}\right) \right| \tag{VI-5-293}$$

- Equations VI-5-292 and VI-5-293 show that the two force components vary with elevation z on the pile and with time t. The inertia force f_i is maximum for sin $(-2\pi t/T) = 1$, which corresponds to $t = -T/4$ for linear wave theory. Thus, the maximum inertia force on the pile occurs $T/4$ seconds before the passage of the wave crest that occurs at $t = 0$ (see Equation VI-5-289). The maximum value of the drag force component f_D coincides with passage of the wave crest at $t = 0$.

- The magnitude of the maximum inertia force per unit length of pile varies with depth the same as the horizontal acceleration component (Equation VI-5-291). The maximum value occurs at the swl ($z = 0$) and decreases with depth. The same trend is true for the maximum drag force per unit length of pile except the decrease with depth is more rapid because the depth attenuation factor (cosh $[2\pi(z+d)/L\}$/cosh$[2\pi d/L]$) is squared in Equation VI-5-293.

- The total time-varying force and the time-varying moment acting about the mudline is found for linear wave theory by integrating Equations VI-5-283 and VI-5-284 between the bottom and the swl ($z = 0$) using the expressions for f_i and f_D given by Equations VI-5-292 and VI-5-293, respectively. The integration results in total force and moment components given by Equations VI-5-285 through VI-5-288 with values of the dimensionless parameters K_i, K_D, S_i, and S_D given by

$$K_i = \frac{1}{2} \tanh\left(\frac{2\pi d}{L}\right) \sin\left(-\frac{2\pi t}{T}\right) \tag{VI-5-294}$$

$$\begin{aligned} K_D &= \frac{1}{8}\left(1 + \frac{4\pi d/L}{\sinh[4\pi d/L]}\right) \cos\left(\frac{2\pi t}{T}\right) \left| \cos\left(\frac{2\pi t}{T}\right) \right| \\ &= \frac{1}{4} n \cos\left(\frac{2\pi t}{T}\right) \left| \cos\left(\frac{2\pi t}{T}\right) \right| \end{aligned} \tag{VI-5-295}$$

$$S_i = 1 + \frac{1 - \cosh[2\pi d/L]}{(2\pi d/L)\sinh[2\pi d/L]} \tag{VI-5-296}$$

$$S_D = \frac{1}{2} + \frac{1}{2n}\left(\frac{1}{2} + \frac{1 - \cosh[4\pi d/L]}{(4\pi d/L)\sinh[4\pi d/L]}\right) \tag{VI-5-297}$$

where

$$n = \frac{C_g}{C} = \frac{1}{2}\left(1 + \frac{4\pi d/L}{\sinh[4\pi d/L]}\right) \tag{VI-5-298}$$

- The maximum values for total inertia force and moment are found by taking $t = -T/4$ in Equations VI-5-294 and VI-5-296, respectively. Likewise, the maximum values for total drag force and moment are found by taking $t = 0$ in Equations VI-5-295 and VI-5-297, respectively. A conservative design approach would be to sum the individual maximum inertia and drag components that occur during a wave cycle to get total maximum force and moments. However, the individual maximums

do not occur simultaneously, so the real maximum total force and moment wil be somewhat less. The correct method is to calculate the time-varying sum of inertia and drag components, and then use the maximum sum that occurs over the wave cycle. The time at which the maximum occurs may vary depending on the selected values for C_M and C_D.

- Although linear wave theory provides a nice closed-form solution for forces and moments on slender cylindrical piles, in practice the hydrodynamics associated with the steeper design wave conditions will not be well predicted by linear wave theory. Even more critical is the fact that linear theory provides no estimate of the force caused by that portion of the wave above the swl, an area where the horizontal velocities and accelerations are the greatest. An ad hoc adjustment is to assume a linear force distribution having a maximum value of force estimated at the still-water line and a value of zero at the crest location of the linear wave (H/2 above the swl). Most likely, the design wave will be nonlinear with steep wave crests and with much of the wave height above the swl, and it would be well advised to use an appropriate nonlinear wave theory in the force and moment calculation.

(b) Nonlinear wave theory.

- Design conditions for vertical cylindrical piles in coastal waters will most likely consist of nonlinear waves characterized by steep crests and shallow troughs. For accurate force and moment estimates, an appropriate nonlinear wave theory should be used to calculate values of u and du/dt corresponding to the design wave height, wave period, and water depth.

- The variation of f_i and f_D with time at any vertical location on the pile can be estimated using values of u and du/dt from tables such as Stoke's fifth-order wave theory (Skjelbriea et al. 1960) or stream-function theory (Dean 1974). Computer programs based on higher order monochromatic wave theories may be available to ease the task associated with using tabulated wave kinematics.

- The separate total maximum inertia force and moment and total drag force and moment on a vertical cylindrical pile subjected to nonlinear waves can be estimated using Equations VI-5-285 through VI-5-288. Values for K_i, K_D, S_i, and S_D in Equations VI-5-285 - VI-5-288 are given by K_{im}, K_{Dm}, S_{im}, and S_{Dm}, respectively, in the nomograms shown in Figures VI-5-126 through VI-5-129. (Note: In the nomograms the subscript m is used to denote maximum.) These nomograms were constructed using stream-function theory (Dean 1974), and they provide the maximum total force and total moment for the inertia and drag components considered separately rather than the combined total force and moment. The curves in Figures VI-5-126 to VI-5-129 represent wave height as a fraction of the breaking wave height. For example, curves labeled 1/2 H_b represent H/H_b = 1/2. Breaking wave height is obtained from Figure VI-5-130 for values of d/gT^2 using the curve labeled Breaking Limit.

- For linear waves, the maximum inertia force occurs at t = -T/4 and the maximum drag force occurs at t = 0. However, for nonlinear waves the times corresponding to maximum inertia and drag forces are phase dependent and not separated by a constant quarter wavelength as in linear wave theory.

- The total maximum force F_m, where the sum of the inertia and drag components is maximum, can be estimated using Figures VI-5-131 to VI-5-134. These figures were also constructed using stream-function theory. Figure selection is based on the nondimensional parameter

$$W = \frac{C_M D}{C_D H} \qquad \text{(VI-5-299)}$$

and the drawn curves give values of φ_m corresponding to the known parameters H/gT^2 and d/gT^2.

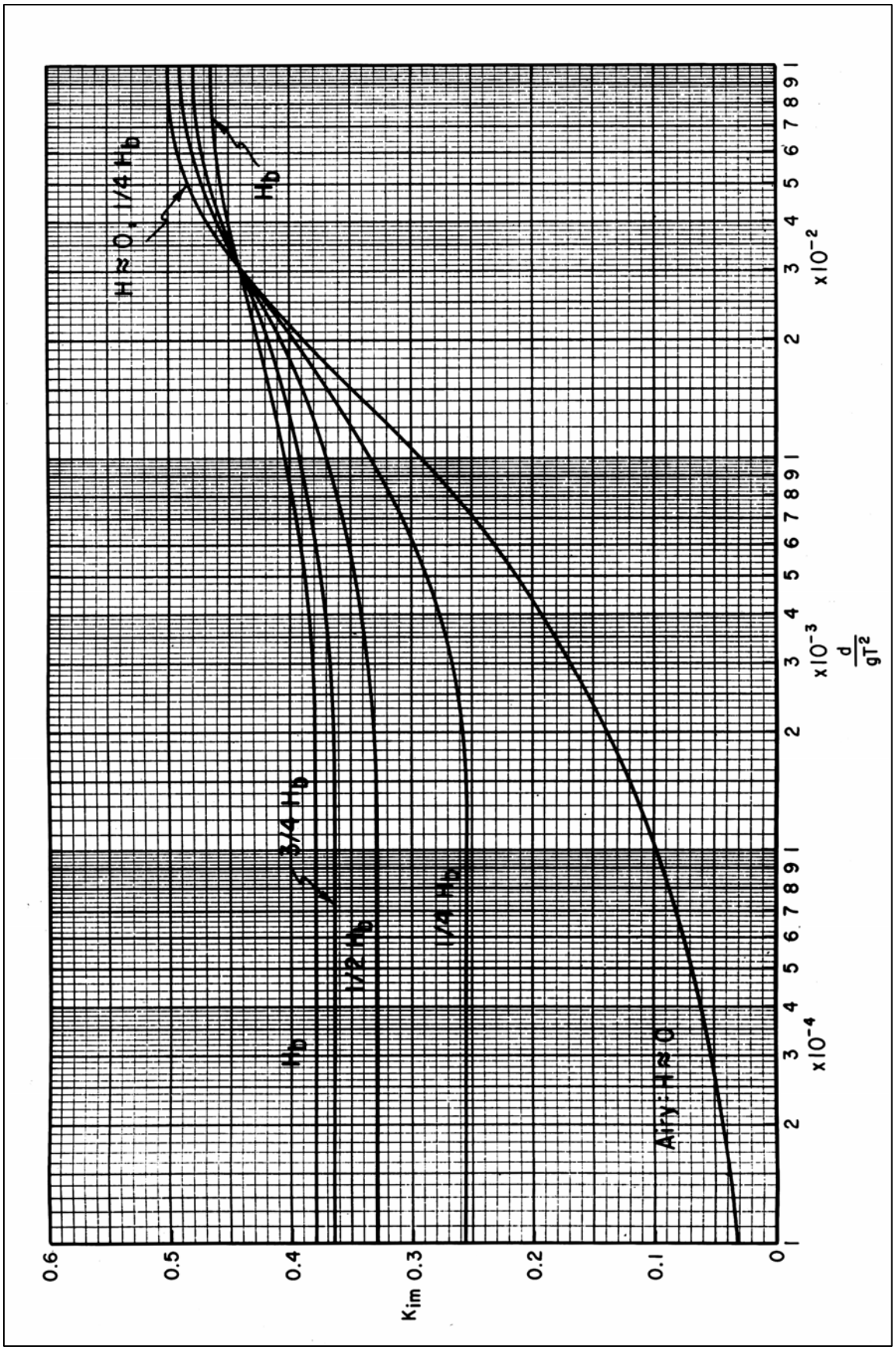

Figure VI-5-126. K_{im} **versus relative depth,** d/gT^2

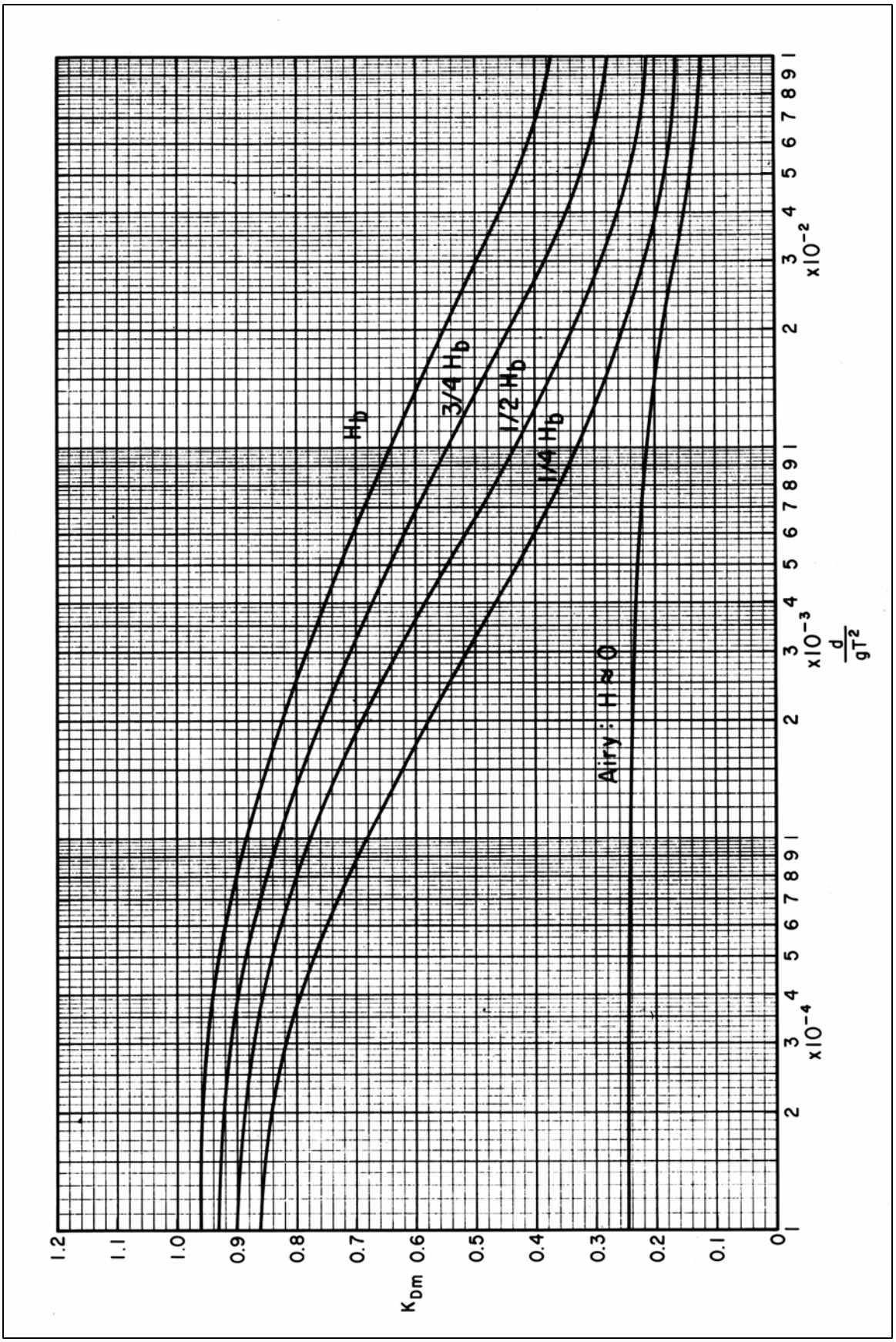

Figure VI-5-127. K_{Dm} versus relative depth, d/gT^2

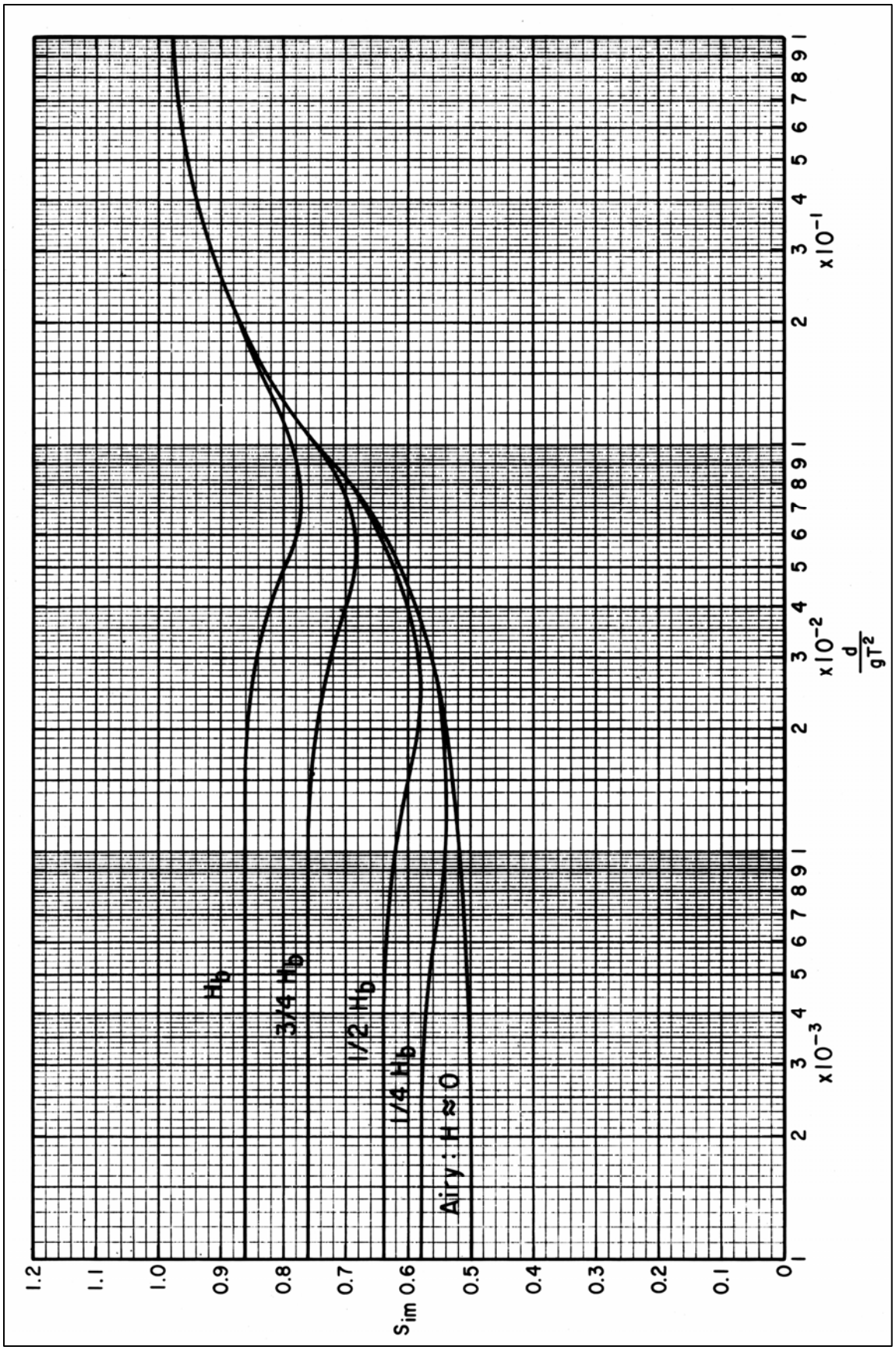

Figure VI-5-128. Inertia force moment arm S_{im} versus relative depth, d/gT^2

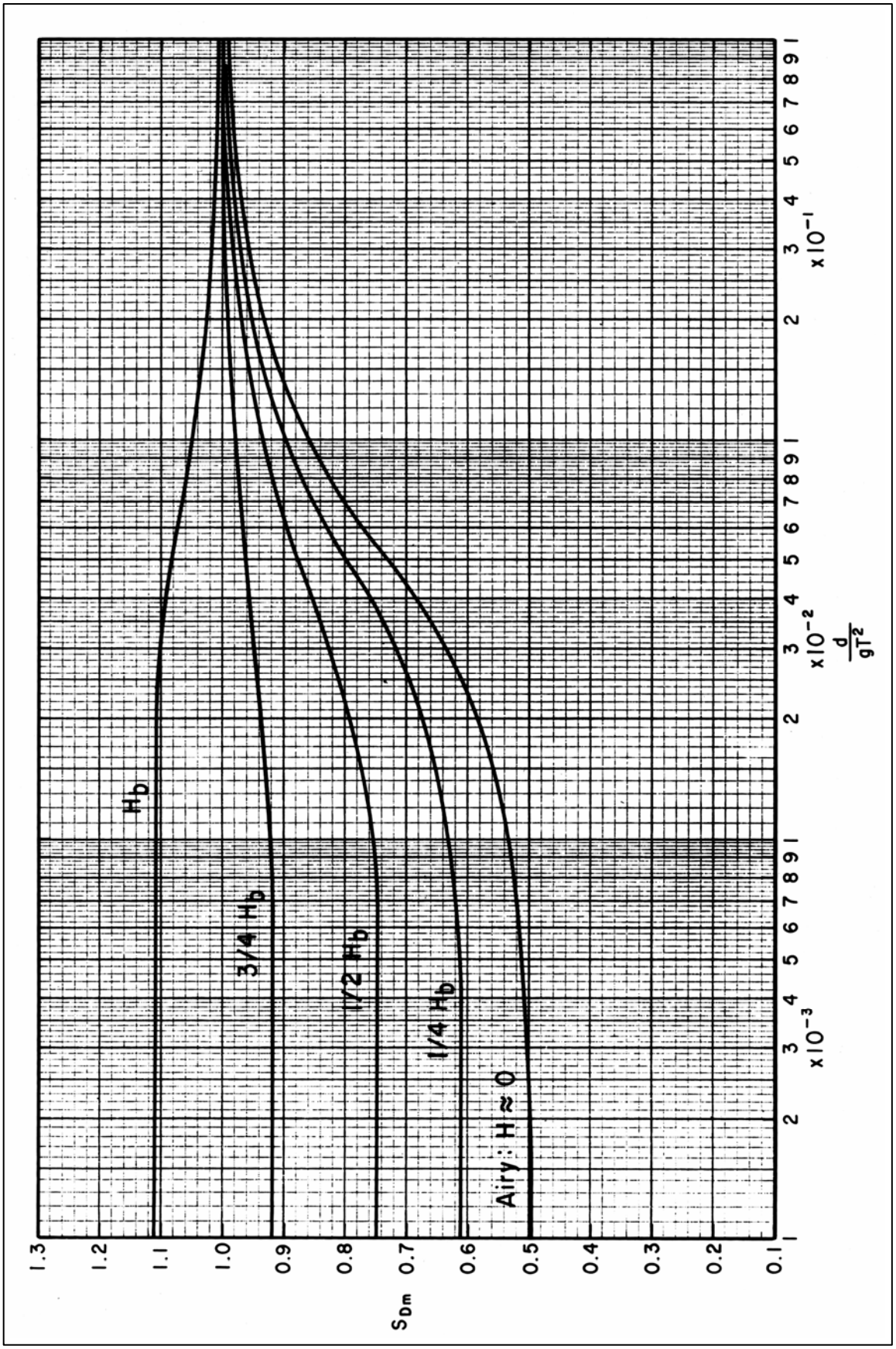

Figure VI-5-129. Drag force moment arm S_{Dm} versus relative depth, d/gT^2

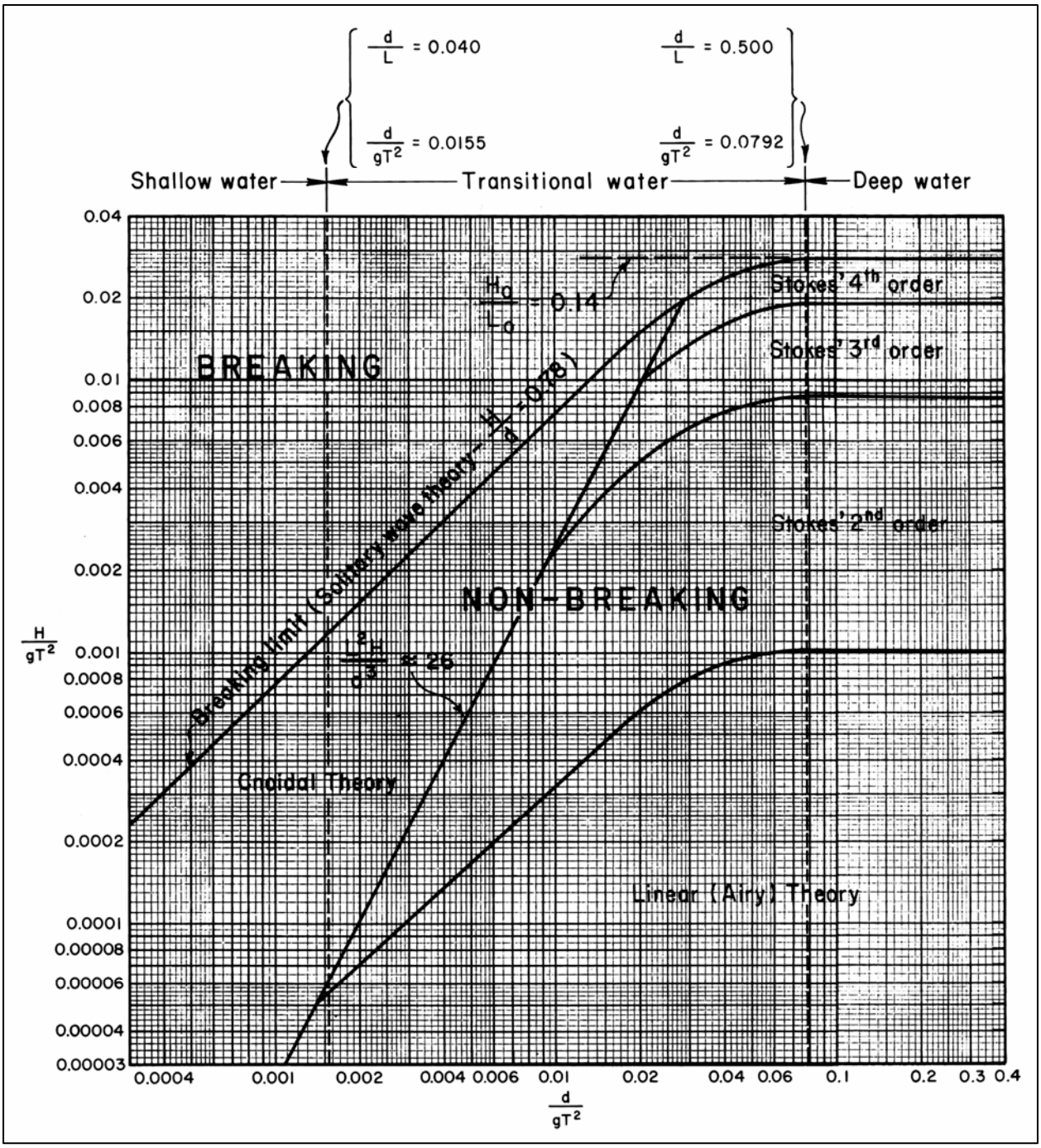

Figure VI-5-130. Breaking wave height and regions of validity of various wave theories

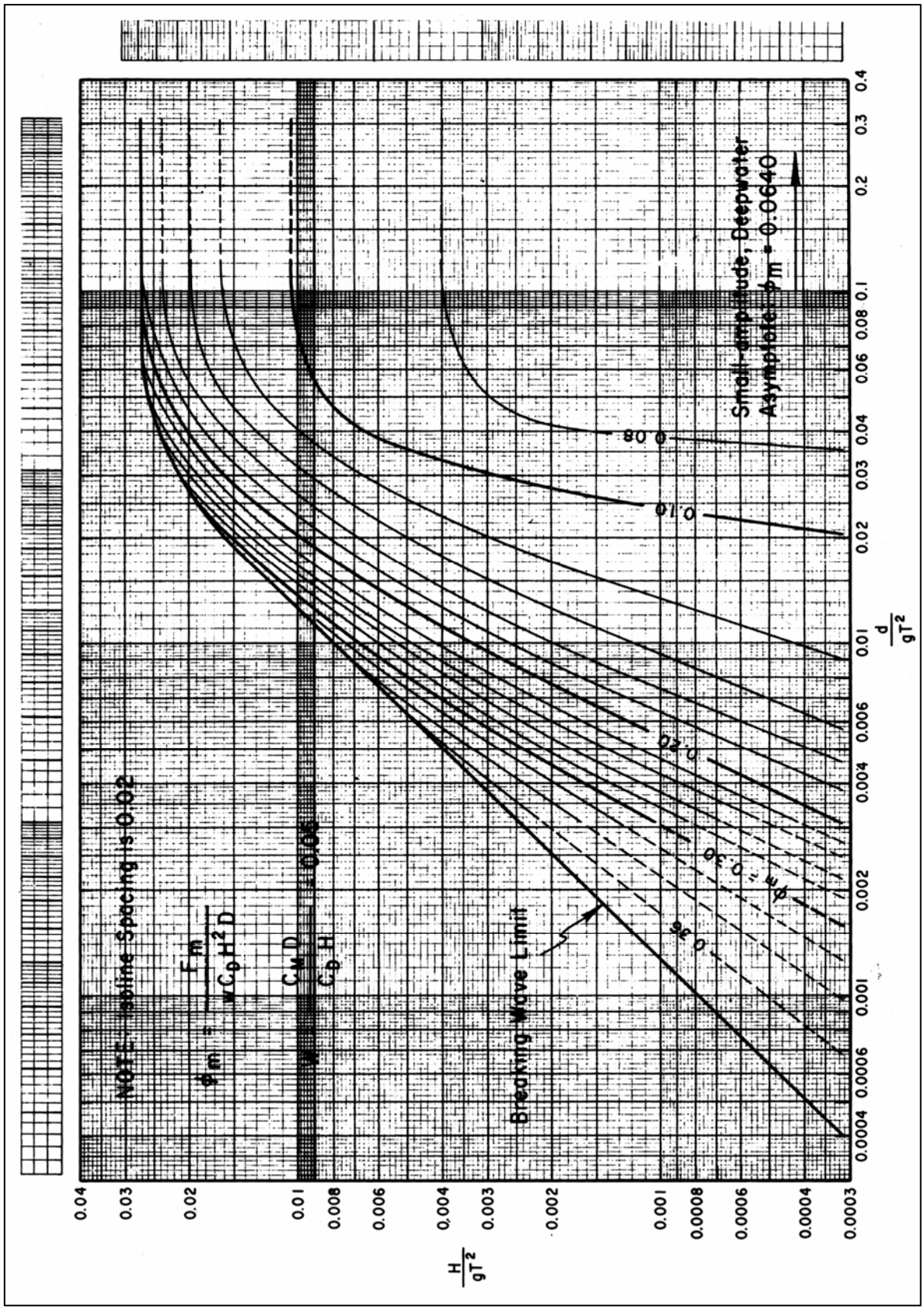

Figure VI-5-131. Isolines of φ_m versus H/gT^2 and d/gT^2 (W = 0.05)

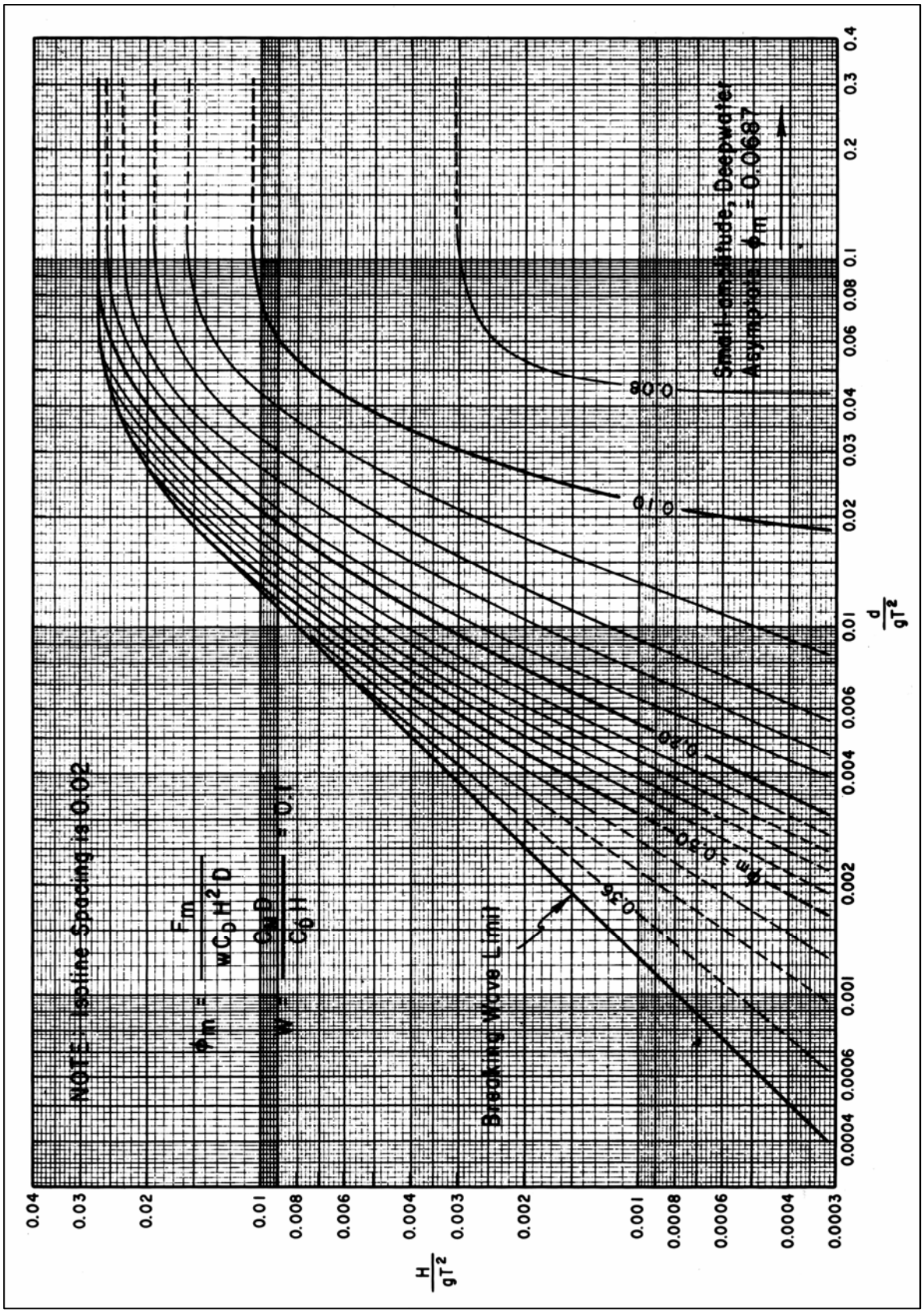

Figure VI-5-132. Isolines of φ_m versus H/gT^2 and d/gT^2 (W = 0.10)

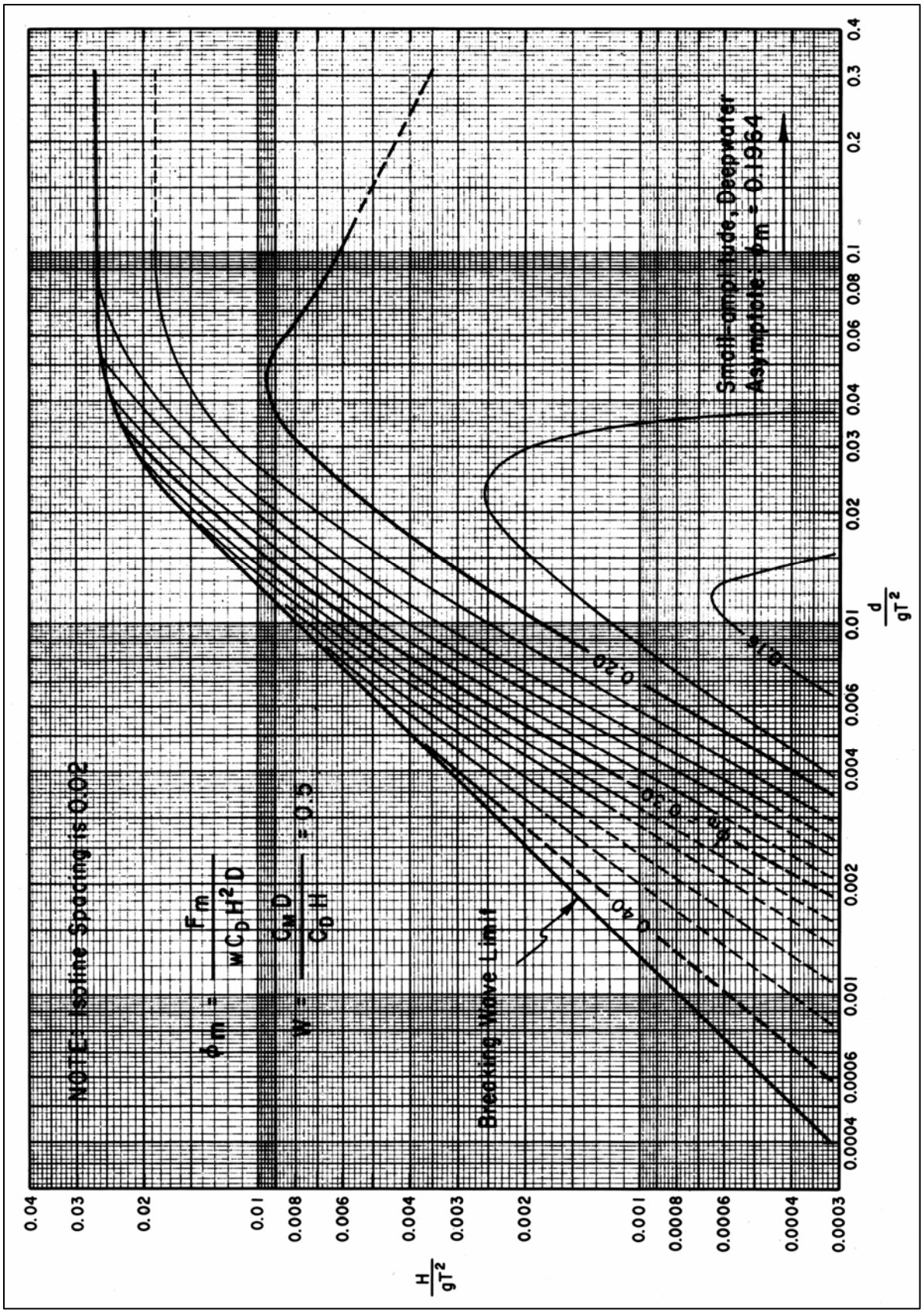

Figure VI-5-133. Isolines of φ_m versus H/gT^2 and d/gT^2 ($W = 0.50$)

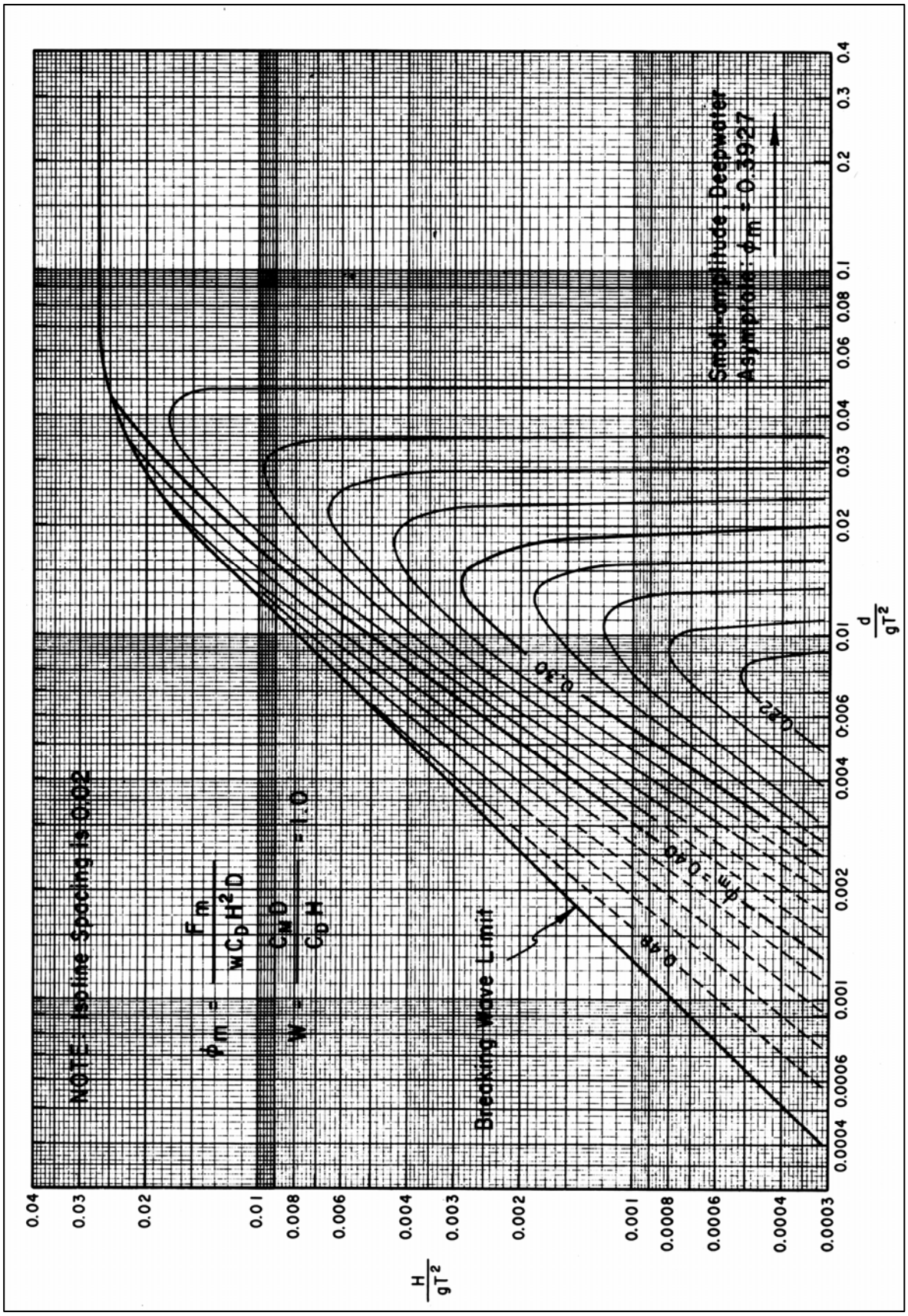

Figure VI-5-134. Isolines of φ_m versus H/gT^2 and d/gT^2 ($W = 0.10$)

- The maximum force is calculated as

$$F_m = \phi_m C_D \rho g H^2 D \qquad \text{(VI-5-300)}$$

- Similarly, the total maximum moment M_m can be estimated using Figures VI-5-135 through VI-5-138 which were also constructed using stream-function theory. Choice of figure is based on the value of W given by Equation VI-5-299, and values for α_m are corresponding to the parameters H/gT^2 and d/gT^2. The moment about the mudline is given by

$$M_m = \alpha_m C_D \rho g H^2 D d \qquad \text{(VI-5-301)}$$

- For both the total force and total moment calculations, the calculated value of W will likely lie between the values for which the figures are drawn. In this case, determine values of φ_m and α_m from the plots on either side of the W-value, then use linear interpolation to estimate values of φ_m and α_m for the calculated value of W.

- The maximum moment is calculated at the mudline, and the corresponding moment arm is the maximum moment divided by the maximum force, or

$$r_a = \frac{M_m}{F_m} \qquad \text{(VI-5-302)}$$

- If the surrounding soil does not provide any lateral resistance, or if there has been scour around the pile, the effective moment arm must be increased and a new maximum total moment calculated. For example, if the scour depth beneath the surrounding bed is S_m, the modified maximum total moment will be

$$M_{m'} = (r_a + S_m) F_m \qquad \text{(VI-5-303)}$$

- See Part VI-7, "Design of Specific Project Elements," for an example illustrating calculation of forces and moments on a vertical cylinder.

(3) Transverse forces due to eddy shedding.

(a) In addition to drag and inertia forces that act in the direction of wave advance, transverse forces may arise. Transverse forces are caused by vortex or eddy shedding on the downstream side of the pile. Eddies are shed alternately from each side of the pile resulting in a laterally oscillating force. Transverse forces act perpendicular to both wave direction and pile axis, and they are often termed lift forces because they are similar to aerodynamic lift acting on an airfoil.

(b) Laird, Johnson, and Walker (1960) and Laird (1962) studied transverse forces on rigid and flexible oscillating cylinders. In general, lift forces were found to depend on the dynamic response of the structure. For structures with a natural frequency of vibration about twice the wave frequency, a dynamic coupling between the structure motion and fluid motion occurs, resulting in large lift forces. Transverse forces have been observed 4.5 times greater than the drag force. However, for rigid structures a transverse force equal to the drag force is a reasonable upper limit. Larger transverse forces can occur where there is dynamic interaction between the waves and cylindrical pile. The design guidance in this section pertains only to rigid piles.

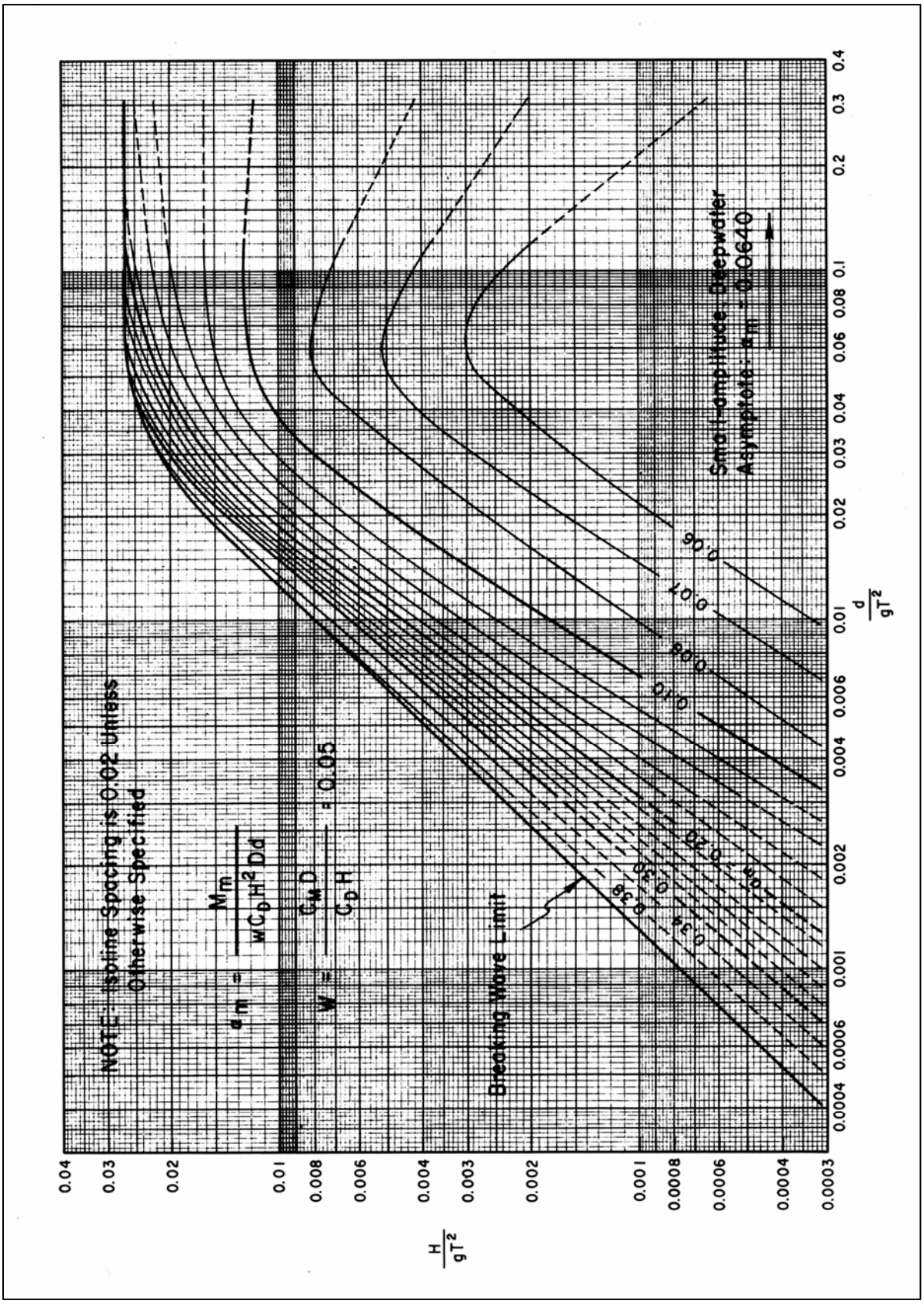

Figure VI-5-135. Isolines of α_m versus H/gT^2 and d/gT^2 ($W = 0.05$)

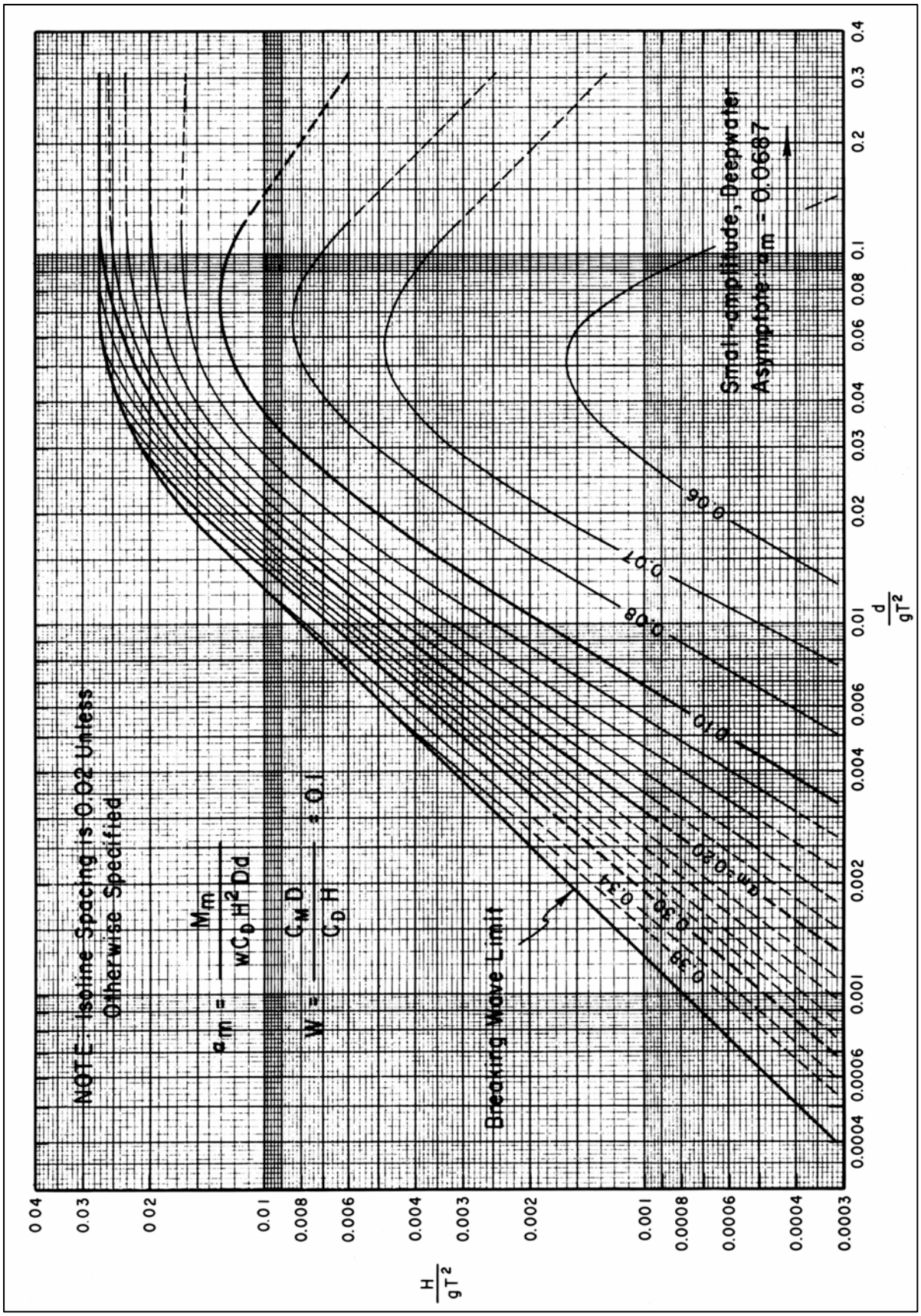

Figure VI-5-136. Isolines of α_m versus H/gT^2 and d/gT^2 ($W = 0.10$)

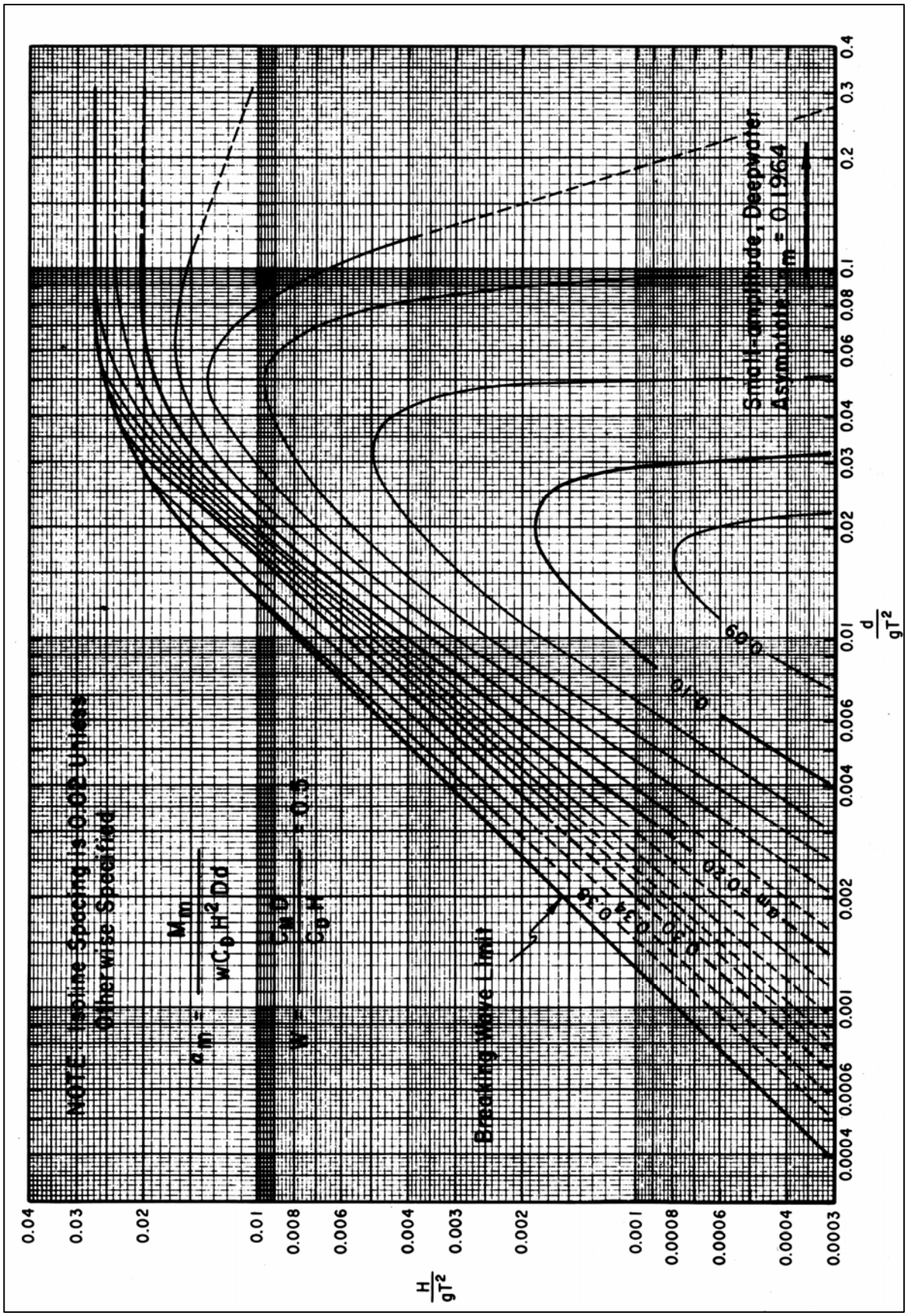

Figure VI-5-137. Isolines of α_m versus H/gT^2 and d/gT^2 ($W = 0.5$)

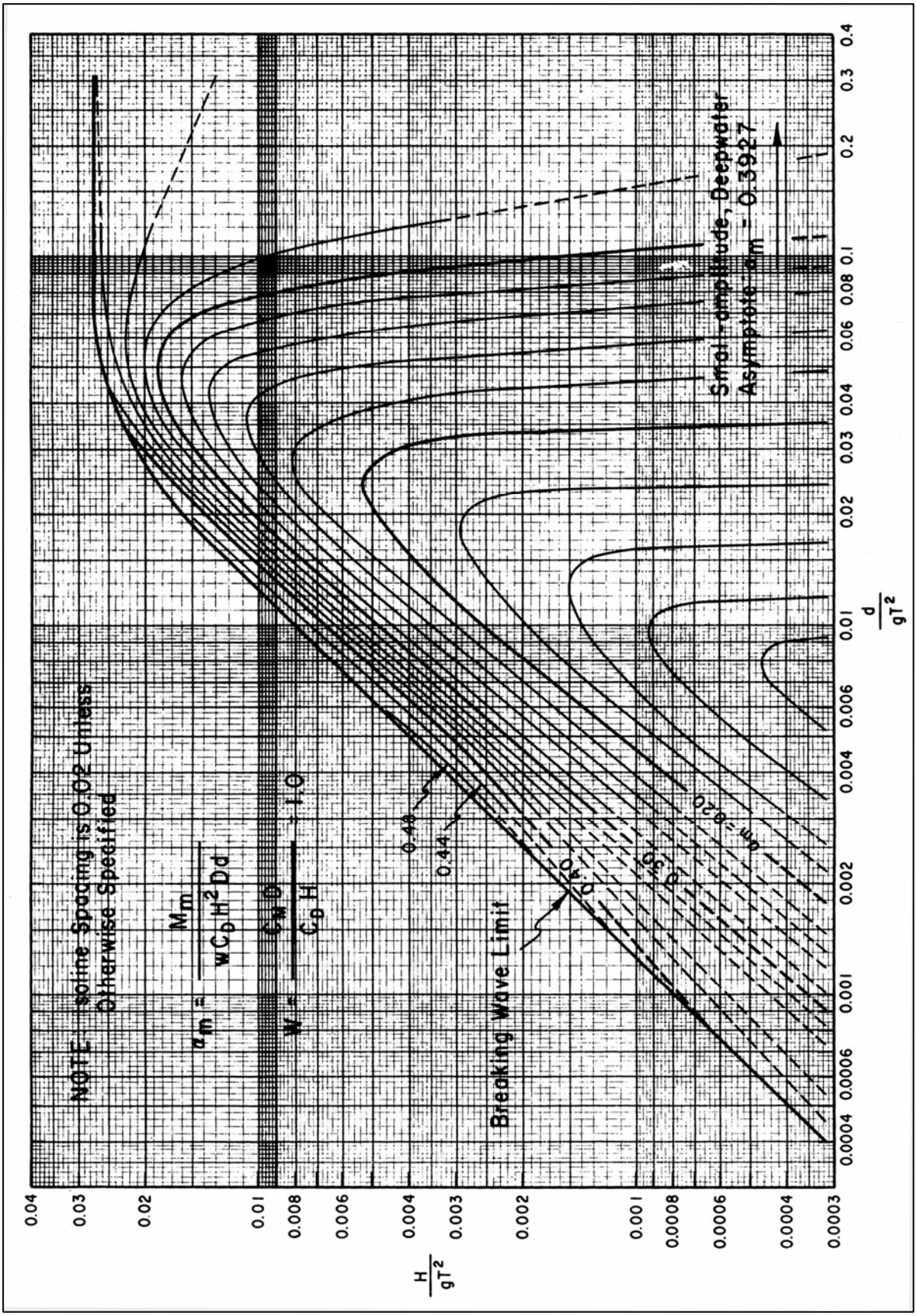

Figure VI-5-138. Isolines of α_m versus H/gT^2 and d/gT^2 ($W = 1.0$)

(c) Chang (1964) found in laboratory investigations that eddies are shed at a frequency that is twice the wave frequency. Two eddies are shed after passage of the wave crest (one on each side of the pile), and two are shed on the return flow after passage of the wave trough. The maximum lift force is proportional to the square of the horizontal wave-induced velocity in much the same way as the drag force. Consequently, for design estimates of the lift force the following equation can be applied.

$$F_L = F_{Lm} \cos 2\theta = C_L \frac{\rho g}{2} D H^2 K_{Dm} \cos 2\theta \qquad \text{(VI-5-304)}$$

where F_L is the time-varying transverse (lift) force, F_{Lm} is the maximum transverse force, θ is the wave phase angle ($\theta = 2\pi x/L - 2\pi t/T$), C_L is an empirical lift coefficient analogous to the drag coefficient in Equation VI-5-286, and K_{Dm} is the dimensionless parameter given in Figure VI-5-127. Chang found that C_L depends on the average Keulegan-Carpenter number given as

$$KC_{ave} = \frac{(u_{\max})_{ave} T}{D} \qquad \text{(VI-5-305)}$$

where $(u_{max})_{ave}$ is the maximum horizontal velocity averaged over the depth. When KC_{ave} is less than 3, no significant eddy shedding occurs and no lift forces are developed. As KC_{ave} increases, C_L increases until it is approximately equal to C_D for rigid piles. Consequently, it must be recognized that: the lift force can represent a major portion of the total force acting on a pile and therefore should not be neglected in the design of the pile.

c. Selection of hydrodynamic force coefficients C_D, C_M, and C_L.

Sarpkaya (1976a, 1976b) conducted an extensive experimental investigation of the inertia, drag, and transverse forces acting on smooth and rough circular cylinders. The experiments were performed in an oscillating U-tube water tunnel for a range of Reynolds numbers up to 700,000 and Keulegan-Carpenter numbers up to 150. Relative roughness of the cylinders k/D varied between 0.002 and 0.02 (where k is the average height of the roughness element). Forces were measured on stationary cylinders, and the corresponding drag and inertia coefficients were determined using a technique of Fourier analysis and least-squares best fit of the Morison equation (Equation VI-5-281) to the measured forces.

The results were presented as plots of the force coefficients versus Keulegan-Carpenter number

$$KC = \frac{u_m T}{D} \qquad \text{(VI-5-306)}$$

for given values of Reynolds number

$$R_e = \frac{u_m D}{\nu} \qquad \text{(VI-5-307)}$$

or the frequency parameter

$$\beta = \frac{R_e}{KC} = \frac{D^2}{\nu T} \qquad \text{(VI-5-308)}$$

In Equations VI-5-306 - VI-5-308 u_m is the maximum horizontal wave velocity, T is the wave period, D is the cylinder diameter, and v is the fluid kinematic viscosity.

Figures VI-5-139 through VI-5-141 present Sarpkaya's (1976a, 1976b) experimental results for the force coefficients C_D , C_M , and C_L for smooth cylinders. In each figure the force coefficient is plotted versus Keulegan-Carpenter number for constant values of Reynolds number (dotted lines) and frequency parameter (solid lines). Drag and inertia force coefficients versus Reynolds number for rough cylinders are plotted on Figures VI-5-142 and VI-5-143, respectively, for selected values of relative roughness *k/D*. Sarpkaya cautioned that the force coefficients were developed for oscillatory flow with zero mean velocity, and it is possible that waves propagating on a uniform current may have different force coefficients.

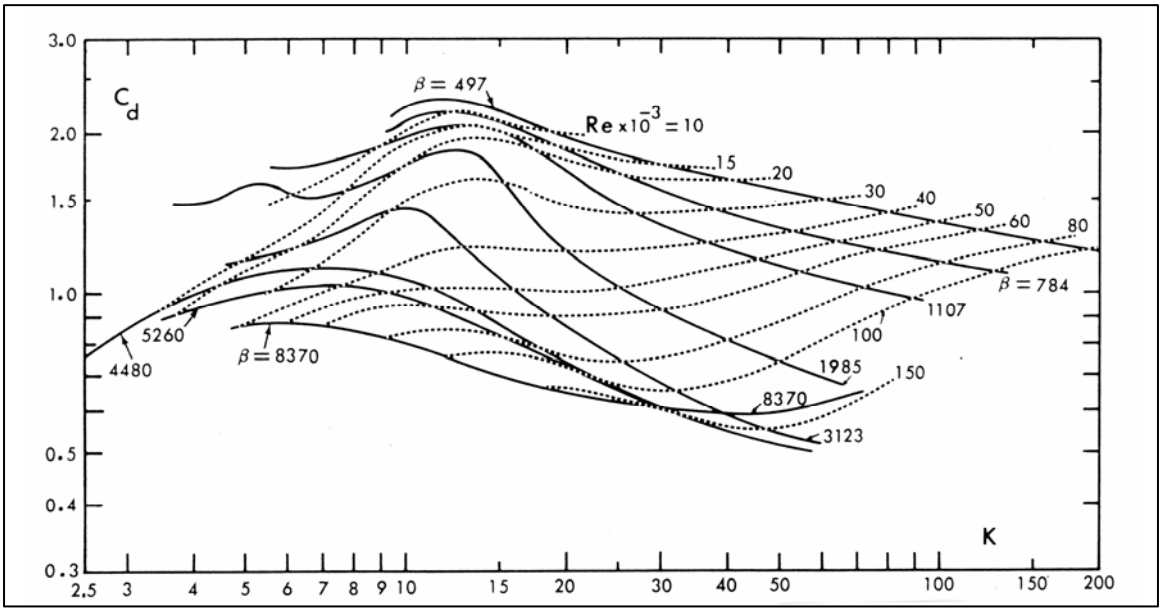

Figure VI-5-139. Drag coefficient C_D as a function of *KC* and constant values of R_e or β for smooth cylinders (from Sarpkaya 1976a)

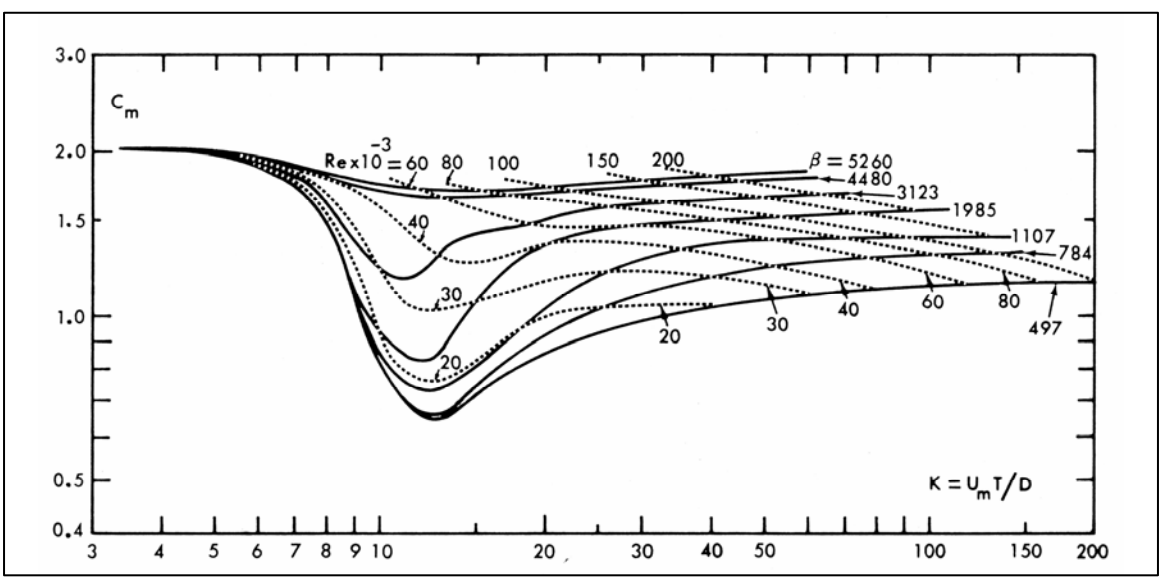

Figure VI-5-140. Drag coefficient C_M as a function of *KC* and constant values of R_e or β for smooth cylinders (from Sarpkaya 1976a)

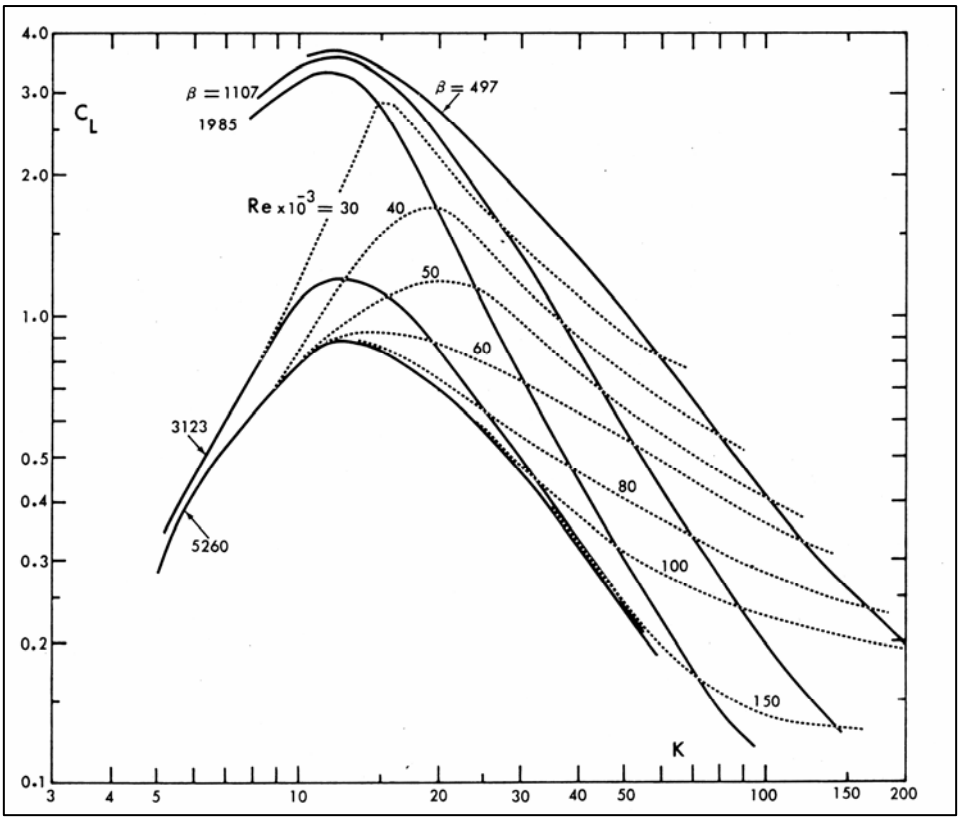

Figure VI-5-141. Lift coefficient C_L as a function of *KC* and constant values of R_e or β for smooth cylinders (from Sarpkaya 1976a)

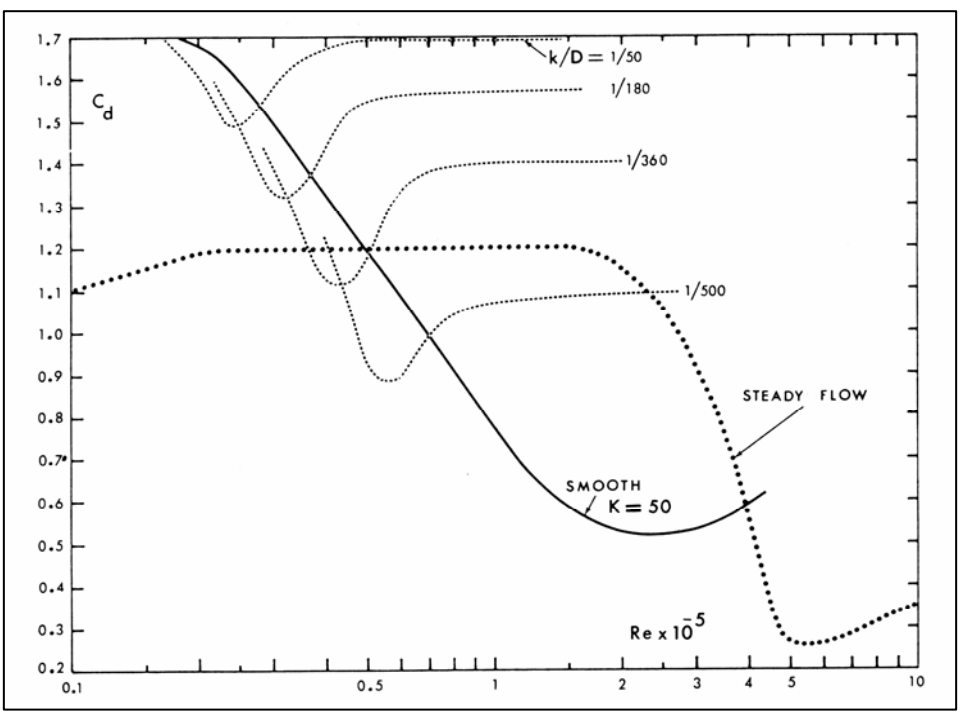

Figure VI-5-142. Drag coefficient C_D as a function of Reynolds number for rough cylinders (from Sarpkaya 1976a)

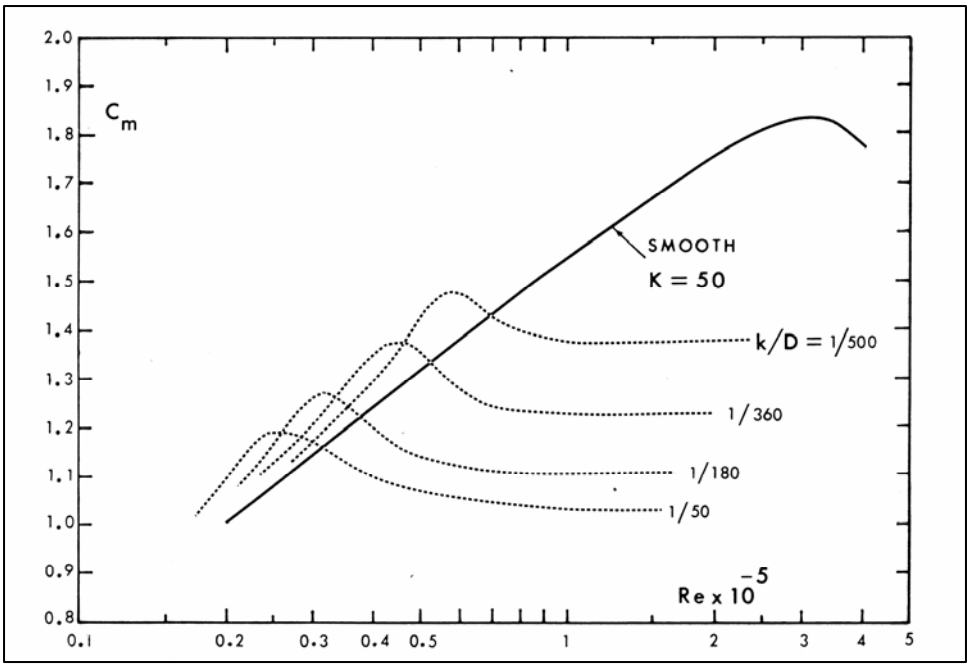

Figure VI-5-143. Drag coefficient C_M as a function of Reynolds number for rough cylinders (from Sarpkaya 1976a)

The force coefficients given in Figures VI-5-139 through VI-5-143 should give reasonable force estimates when used with the design figures based on stream function theory given in the previous section. However, the design engineer should be aware of the limitations of assuming the force coefficients are constant over the water depth and throughout the wave cycle.

Sarpkaya's experimental apparatus gave uniform values of Reynolds number and Keulegan-Carpenter number over the entire test pile. For a vertical pile exposed to waves, the maximum horizontal velocity will vary from its largest value at the sea surface to a somewhat smaller value near the bottom. Consequently, both R_e and KC will vary over the depth of the pile. For design purposes, it is reasonable to calculate R_e and KC based on the average value of u_m over the water depth in shallow water because the variation will not be too significant. In deeper water it may be wise to investigate the variation of force coefficients with depth to determine if using R_e and KC based on average u_m is appropriate.

Sarpkaya's experimental data do not cover the range of Reynolds numbers likely to be encountered with bigger waves and larger pile diameters. For larger calculated Reynolds numbers use the following guidance that has been repeated from the old *Shore Protection Manual* (1984).

$$C_D = \begin{cases} 1.2 - \dfrac{(R_e - 2(10)^5}{6(10)^5} & for\ \ 2(10)^5 < R_e - 5(10)^5 \\ 0.7 & for\ \ 5(10)^5 < R_e \end{cases}$$

$$C_M = \begin{cases} 2.5 - \dfrac{R_e}{5(10)^5} & for\ \ 2.52(10)^5 < R_e - 5(10)^5 \\ 1.5 & for\ \ 5(10)^5 < R_e \end{cases}$$

Bear in mind the above recommendations for higher Reynolds number are based on older experimental results, and more accurate estimates might be available from the offshore engineering literature for critical applications.

d. Safety factors in pile design.

Before the pile is designed or the foundation analysis is performed, a safety factor is usually applied to calculated forces. Reasons for uncertainty to the design include approximations in applying the wave theory, estimated values for the force coefficients, potential loss of pile strength over time, and the probability that the design wave will be exceeded during the life of the structure.

The following recommendations for safety factors are offered as general rules of thumb. In situations where pile failure could lead to loss of life or catastrophic failure of supported infrastructure, safety factors should be increased.

(a) When the design wave has low probability of occurrence, it is recommended that a safety factor of 1.5 be applied to calculated forces and moments that are to be used as the basis for structural and foundation design.

(b) If the design wave is expected to occur frequently, such as in depth-limited situations, a safety factor of at least 2.0 should be applied to the calculated forces and moments.

In addition to the safety factor, changes occurring during the expected life of the pile should be considered in design. Such changes as scour about the pile base and added pile roughness due to marine growth may be important.

The design procedure presented in the previous sections is a static procedure; forces are calculated and applied to the structure statically. The dynamic nature of forces from wave action must be considered in the design of some offshore structures. When a structure's natural frequency of oscillation is such that a significant amount of energy in the wave spectrum is available at that frequency, the dynamics of the structure must be considered. In addition, stress reversals in structural members subjected to wave forces may cause failure by fatigue. If fatigue problems are anticipated, the safety factor should be increased or allowable stresses should be decreased. Evaluation of these considerations is beyond the scope of this manual.

Corrosion and fouling of piles also require consideration in design. Corrosion decreases the strength of structural members. Consequently, corrosion rates over the useful life of an offshore structure must be estimated and the size of structural members increased accordingly.

Fouling of a structural member by marine growth increases the roughness and effective diameter of the pile and also changes the values of the force coefficients. The increased diameter must be carried through the entire design procedure to determine forces on the fouled member.

e. Other considerations related to forces on slender cylindrical piles.

(1) Wave forces on pile groups. For a group of piles supporting a structure such as a platform or pier, the methods given in the previous sections can be used provided the piles are sufficiently separated so that flow around one pile does not influence the flow around adjacent piles. One approach is to assume waves are long crested and of permanent form. Given the relative orientation of the piles to each other and to the incoming wave, forces can be estimated on each pile at different times during the wave passage. Typically, the maximum force on individual piles occurs at different times unless all the piles are parallel to the wave crest.

Therefore, numerous calculations throughout the wave passage are needed to determine the worst loading on the overall structure. Because the tops of the piles are connected by the superstructure, and the connections may provide some rigidity; it may be necessary to analyze the pile group as a frame.

As the distance between piles becomes small relative to the wavelength, maximum forces and moments on pile groups may be conservatively estimated by summing the maximum forces and moments on each pile.

The assumption that piles are unaffected by neighboring piles is not valid when the distance between piles is less than about three times the pile diameter. Chakrabarti (1991) presented design graphs giving maximum force on a pile in a linear pile group (piles aligned in a row) as a function of Keulegan-Carpenter number and relative separation distance *S/D* where *S* is the distance between center lines of adjacent piles. Graphs were provided for pile groups consisting of two, three and five piles with waves approaching parallel and perpendicular to the line of piles. Graphs were also given for estimating C_D, C_M, and C_L for pile groups of three and five piles exposed to waves parallel and perpendicular to the pile line.

(2) Wave forces on nonvertical piles. Forces and moments on nonvertical cylindrical piles can be estimated using Morison's equation (Equation VI-5-281) where the values for velocity *u* and acceleration *du/dt* are given as the velocity and acceleration components perpendicular to the pile. Calculations will need to be performed using an appropriate wave theory along with the force coefficients given in Part VI-5-7*c*, "Selection of hydrodynamic force coefficients C_D, C_M, *and* C_L." Do not use the curves provided in design Figures VI-5-126 through VI-5-129 and VI-5-131 through VI-5-134 because these figures are only for vertical piles. For nonvertical piles, the pile self weight (immersed and above water) will contribute to the overturning moment and must be included in the calculation.

(3) Broken wave forces. Forces resulting from action of broken waves on piles are much smaller than forces due to breaking waves. When pile-supported structures are constructed in the surf zone, lateral forces from the largest wave breaking on the pile should be used for design. Although breaking-wave forces in the surf zone are great per unit length of pile, the pile length actually subjected to wave action is usually short. Hence, the resulting total force and moment are small. Pile design in the surf zone is usually governed primarily by vertical loads acting along the pile axis.

VI-5-8. Other Forces and Interactions

a. Impact forces. Impact force loading on coastal projects occurs when waves or solid objects collide with typically stationary coastal structure elements. Only solid body impacts are discussed in this section. Impact loads between shifting concrete armor units are discussed in Part VI-5-3*c*, "Structural integrity of concrete armor units."

Certain coastal structures such as thin-walled flood barriers, sheet-pile bulkheads, mooring facilities, coastal buildings, or other infrastructure may be subject to impact damage by solid objects carried by waves, currents, or hurricane-force winds. During severe storms, high winds may propel small pleasure craft, barges, and floating debris which can cause significant horizontal impact loads on structures. Likewise, floating ice masses can also cause great impact loads. Impact loads are an important consideration in design of vessel moorings and fendering systems.

Designing a structure to resist impact loads during extreme events is difficult because of uncertainty associated with impact speed and duration. In situations where impact damage by large floating objects could cause catastrophic loss, it may be prudent to limit adjacent water depth by constructing sloping rubble-mound protection fronting the structure or by placing submerged breakwaters seaward of the structure to ground large floating masses and eliminate the hazard.

Impact forces are evaluated using impulse-momentum and energy considerations found in textbooks on fundamental dynamics. However, application of these principles to particular impact problems is difficult unless reliable estimates can be made of object mass (including added mass in water), the mass initial and final velocities, duration of impact loading, and distribution of impact force over time. In addition, some evaluation must be made on whether the collision of the floating object with a coastal structure results in purely elastic behavior in which momentum is conserved, purely plastic impact with all the kinetic energy of the impact being absorbed, or some combination of the two.

Fendering systems in ports and harbors are designed to absorb low-velocity impacts by vessels during docking maneuvers and seiching motions. Design of fendering systems is adequately covered by numerous textbooks and design standards. Examples of typical design references in the coastal engineering literature include Quinn (1972) and Costa (1973). The modes of kinetic energy absorption by fendering systems were studied theoretically by Hayashi and Shirai (1963). Otani, Horii, and Ueda (1975) presented field measurements related to absorption of impact kinetic energy of 50 large tankers. They observed that most berthing velocities are generally below 6 cm/s, and that measured impact energy was substantially larger than calculated using the design standards that existed at that time. Kuzmanovic and Sanchez (1992) discussed protective systems for bridge piers and pilings, and they gave procedures for accessing the equivalent static force acting on bridge piers due to vessel impacts.

b. Ice forces. A description of ice loading and how it may impact various types of coastal structures in the context of site-specific design criteria is given in Part VI-3-5, "Ice." Other general references include Chen and Leidersdorf (1988); Gerwick (1990); and Leidersdorf, Gadd, and Vaudrey(1996). The following section presents methods for calculating ice forces under specific loading conditions.

(1) Horizontal ice forces.

(a) Solid ice forces.

- Large horizontal forces can result when solid sheet ice, or large chunks of solid ice that have broken free, come in contact with vertical-front coastal structures. Most ice sheets are large enough that impact forces are limited by ice failure in the weakest mode permitted by the mechanics of interaction as the structure penetrates the ice, i.e., crushing, splitting, shear, or bending. For smaller ice blocks or wide structures, the maximum impact force may be limited by the kinetic energy available at the moment of impact (HQUSACE 1982). Ice impact calculations should be based on impulse-momentum considerations, but such calculations will be difficult because of uncertainty in estimating a value for ice block velocity.

- Wind and water current drag acting on large floating blocks of ice press the ice blocks against structures creating large pressures at the points of contact. The force due to drag on a block of ice can be calculated for wind and water currents using the following formula (PIANC 1992)

$$F_d = C_{sf} \rho A (u - u_i)^2 \qquad \text{(VI-5-309)}$$

where

C_{sf} = coefficient of skin friction between wind and ice or water and ice (see Table VI-5-89)

ρ = fluid density (air or water)

Table VI-5-89
Values of Skin Friction Coefficient, C_{sf} (PIANC 1992)

	Smooth Ice	Rough (Pack) Ice
Wind drag	0.001 - 0.002	0.002 - 0.003
Water drag	0.002 - 0.004	0.005 - 0.008

A = horizontal area of ice sheet

u = fluid velocity (10 m above ice for air or 1 m below ice for water)

u_i = velocity of ice in the direction of u

- Separate drag calculations should be performed for both wind and water currents with the results treated as vector forces on the ice mass. Because drag force is directly proportional to ice surface area, larger ice sheets will exert greater forces.

- Once an ice sheet has come to rest against a structure, u_i is zero, and the total drag force can be calculated. Intact ice sheets should be treated as solid bodies with the resultant loads vectorially distributed among the structure/ice contact points using force and moment balance. The total force may be somewhat uniformly distributed along a lineal vertical wall. However, if the ice block comes in contact at only a few discrete points, the contact pressure may be very large. In these cases, the calculated force due to drag may exceed the force necessary for local crushing of the ice, in which case the local crushing strength becomes the limiting force applied to the structure.

(b) Localized ice crushing forces.

- The limiting ice force on a vertical structure is determined by the crushing failure strength of the ice in compression. A theoretical expression for the horizontal ice crushing force was given in Korzhavin in a 1962 Russian publication (Ashton 1986) as

$$\frac{F_c}{bh_i} = m\,I\,k\,x\,\sigma_c \qquad \text{(VI-5-310)}$$

where

F_c = horizontal crushing force

b = structure horizontal width or diameter

h_i = thickness of ice sheet

m = plan shape coefficient

I = indentation coefficient

k = contact coefficient

x = strain rate function

σ_c = ice compressive failure strength in crushing

- This formula is usually applied to piles and pier structures rather than long vertical walls. The plan shape coefficient, *m*, is 1.0 for flat surfaces, 0.9 for circular piles, and $0.85[\sin(\beta/2)]^{1/2}$ for wedge-shaped structures having a wedge angle of β. The indentation coefficient, *I*, has been found experimentally to be a function of the aspect ratio, b/h_i, and it is usually presented in graphical form. The contact coefficient, *k*, is a function of ice velocity and width of structure, and it varies between values of 0.4 to 0.7 for ice velocities between 0.5 and 2.0 m/s. The strain rate coefficient is also a function of ice speed. Ashton (1986) provided further details about the theoretical development of Equation VI-5-310 and its associated coefficients.

- In a Russian publication, Afansev (Ashton 1986) combined the coefficients *I*, *k*, and *x* of Equation VI-5-310 into a single coefficient, *C*, giving the formula

$$\frac{F_c}{bh_i} = C\, m\, \sigma_c \qquad \text{(VI-5-311)}$$

- Afansev established the following empirical relationship for *C* based on model tests using laboratory-grown, saline ice.

$$C = \left(5\frac{h_i}{b} + 1\right)^{1/2} \qquad for\ \ 1 < \frac{b}{h_i}$$
$$C = 4.17 - 1.72\left(\frac{b}{h_i}\right) \qquad for\ \ 0.1 < \frac{b}{h_i} < 1 \qquad \text{(VI-5-312)}$$

- The lower formula in Equation VI-5-312 is a linear interpolation as recommended in Ashton (1986).

- In Equation VI-5-311 values of the shape coefficient are the same as given for the Korzhavin formula (Equation VI-5-310).

- The Canadian Standards Association Bridge Code (Canadian Standards Association 1978) recommended an even more simplified version of Equation VI-5-310 given by

$$\frac{F_c}{bh_i} = \sigma_c \qquad \text{(VI-5-313)}$$

using the range of values for sheet ice compressive crushing strength shown in Table VI-5-90. Equation VI-5-313 and the crushing strength values of Table VI-5-90 were also adopted by the American Association of State Highway and Transportation Officials.

- Use of Equation VI-5-313 implies that the product $C \cdot m = 1$ in Equation VI-5-311, which corresponds to large values of b/h_i. This is a realistic assumption for large bridge piles and piers, but ice crushing forces on smaller diameter piles should be calculating using the appropriate strength values from Table VI-5-90 in Equation VI-5-311.

Table VI-5-90
Values of Effective Ice Crushing Strength, σ_c

Ice Crushing Stress	Environmental Situation
0.7 MPa (100 psi)	Ice breakup occurs at melting temperatures and the ice moves in small pieces that are essentially disintegrated.
1.4 MPa (200 psi)	Ice breakup occurs at melting temperatures, but the ice moves in large pieces that are generally sound.
2.1 MPa (300 psi)	Ice breakup consists of an initial movement of the entire ice sheet or large sheets of sound ice impact piers.
2.8 MPa (400 psi)	Ice breakup occurs with an ice temperature significantly below the melting point and ice movement consists of large sheets.

(c) Thermal ice forces. Equations are available for predicting ice temperature based on an energy balance between the atmosphere and the ice sheet. However, the required parameters (air temperature, air vapor pressure, wind, and cloud cover) needed to calculate thermal expansion are difficult to estimate. Thermal strain is equal to the ice thermal expansion coefficient times the change in ice temperature. For restrained or partially restrained ice sheets a nonlinear, time dependent stress-strain law is used to predict thermal stresses (HQUSACE 1982). Because of stress relaxation due to creep, the rate of thermal change is an important factor; and even a thin snow cover can drastically reduce thermal stresses in ice sheets.

- A design rule-of-thumb for thermal expansion loads per unit horizontal length on dams and other rigid structures is 145 - 220 kN/m (10,000 - 15,000 lbs/ft) (HQUSACE 1982). Movable structures should allow for 73 kN/m (5,000 lbs/ft). These values are based on field measurements.

- Thermal expansion of water frozen between elements of a coastal structure can result in dislocation of individual elements or cracking of armor units making the protection vulnerable to wave attack.

(2) Ice forces on slopes.

(a) Ride-up of ice on slopes.

- When horizontally moving ice encounters a sloping structure, a component of the horizontal force pushes the ice up the slope. This action induces a bending failure in the ice sheet at loads less than required for ice crushing failure. Ashton (1986) showed the derivation of a simple two-dimensional theory for calculating the horizontal force exerted by ice on a sloping structure as illustrated in Figure VI-5-144. (Ashton also included discussion and analysis of the more complex case of ice ride-up on three-dimensional structures).

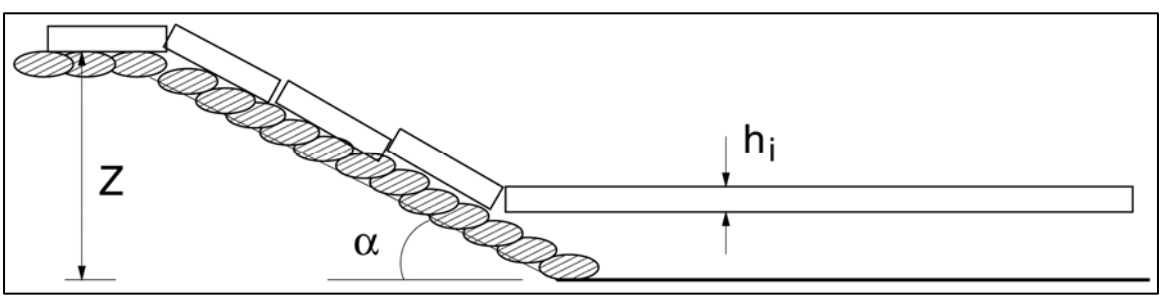

Figure VI-5-144. Ice riding up on structure slope

- For the two-dimensional case the horizontal force per unit width of structure was given by the expression

$$\frac{F_h}{b} = C_1 \sigma_f \left(\frac{\rho_w g h_i^5}{E} \right)^{1/4} + C_2 Z \rho_i g h_i \qquad \text{(VI-5-314)}$$

with

$$C_1 = 0.68 \left[\frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha} \right] \qquad \text{(VI-5-315)}$$

and

F_h = total horizontal force

$$C_2 = \left[\frac{(\sin\alpha + \mu\cos\alpha)^2}{\cos\alpha - \mu\sin\alpha} + \frac{\sin\alpha + \mu\cos\alpha}{\tan\alpha} \right] \qquad \text{(VI-5-316)}$$

b = horizontal width of structure contact zone

h_i = ice sheet thickness

σ_f = flexural strength of ice (0.5 = 1.5 MPa)

ρ_w = water density

ρ_i = ice density (915 = 920 kg/m^3)

E = modulus of elasticity of ice (1,000 = 6,000 MPa)

Z = maximum vertical ice ride-up distance

g = gravitational acceleration

α = structure slope angle relative to horizontal

μ = structure slope friction factor (0.1 = 0.5)

- The first term in Equation VI-5-314 is interpreted as the force necessary to break the ice in bending, and the second term is the force that pushes the ice blocks up the sloping structure. The modulus of elasticity varies from 1,000 MPa for very salty water up to about 6,000 MPa for fresh water (Machemehl 1990). Ashton (1986) warned that this simple two-dimensional theory will be inadequate for narrow structures because the zone of ice failure will be wider than the structure.

- Low values of friction factor (μ = 0.1) are associated with smooth slopes such as concrete or carefully layed block protection, whereas high values (μ = 0.5) are applicable for randomly-placed stone armor, riprap, or filled geotextile bags. For slopes steeper than 1:1, the horizontal ice force increases

rapidly for the higher friction factors, and there is a risk of the dominant failure mode being crushing or buckling rather than bending. Milder slopes with smooth surfaces are much more effective in reducing horizontal ice forces. Croasdale, Allyn, and Roggensack (1988) discussed several additional aspects related to ice ride-up on sloping structures.

- Quick "rough" estimates of horizontal forces on sloping structures can be made using a variation of Equation VI-5-313 as proposed in Ashton (1986), i.e.,

$$\frac{F_h}{b} = K_h h_i \sigma_c \tag{VI-5-317}$$

where K_h is approximated from a curve given in Ashton (1986) by the formula

$$K_h = 1 - 0.654\ f^{0.38} \tag{VI-5-318}$$

with

$$f = \frac{1 - \mu \tan \alpha}{\mu + \tan \alpha} \tag{VI-5-319}$$

and σ_c is the ice compressive strength as given in Table VI-5-90. As slope angle increases, K_h approaches a value of unity which represents failure by crushing. For decreasing slope angles, K_h decreases because of the increasing tendency of the ice to fail in bending. Values of K_h less than 0.2 should never to used in Equation VI-5-317.

(b) Adfreeze loads. When ice that is in contact with a coastal structure is stationary for a sufficient time, the ice will freeze to the structure or its elements. Adfreeze loads result if the ice then moves either horizontally by dynamic ice thrust or vertically due to changing water level. This is more of a problem in lakes with slowly varying water levels than in tidal waters.

- Little guidance is available on adfreeze stresses with adhesion strength varying between 140 kPa to 1050 kPa for freshwater ice (PIANC 1992). Adfreeze may dislodge individual armor stones on rubble-mound slopes creating a weakness in the armor layer. This can be prevented by using oversized stones or interlocking armor on the slope. A survey of riprap structures at Canadian hydropower reserviors concluded that plucking of individual stones frozen to ice could be largely prevented by sizing the riprap median diameter (d_{50}) greater than the expected maximum winter ice thickness (Wuebben 1995).

(3) Vertical ice forces. Ice frozen to coastal structures can create vertical forces due to ice buoyancy effects when water level rises, or by ice weight when water level falls. These vertical forces will persist until the ice sheet fractures due to bending or the adfreeze force is exceeded.

(a) Cylindrical piles.

- In cases where the ice sheet freezes around a pile, forces will be exerted on the pile if the water level rises or falls. A rising water level will lift the ice sheet, and under certain conditions the uplift force on the pile may be sufficient to pull the pile free. Similarly, during falling water levels the weight of the ice sheet will exert a downward force on a pile which may be sufficient to buckle a slender pile.

- Kerr (1975) studied vertical loads on cylindrical piles and presented equations for calculating loads under the conservative assumption that the water level change is rapid enough to assure elastic ice behavior before failure. A closed-form solution to the governing equation was obtained in terms of Bessel functions, and Kerr presented a numerically evaluated solution in graphical form as shown on Figure VI-5-145. The graphical solution is dimensional, and it has the functional form of

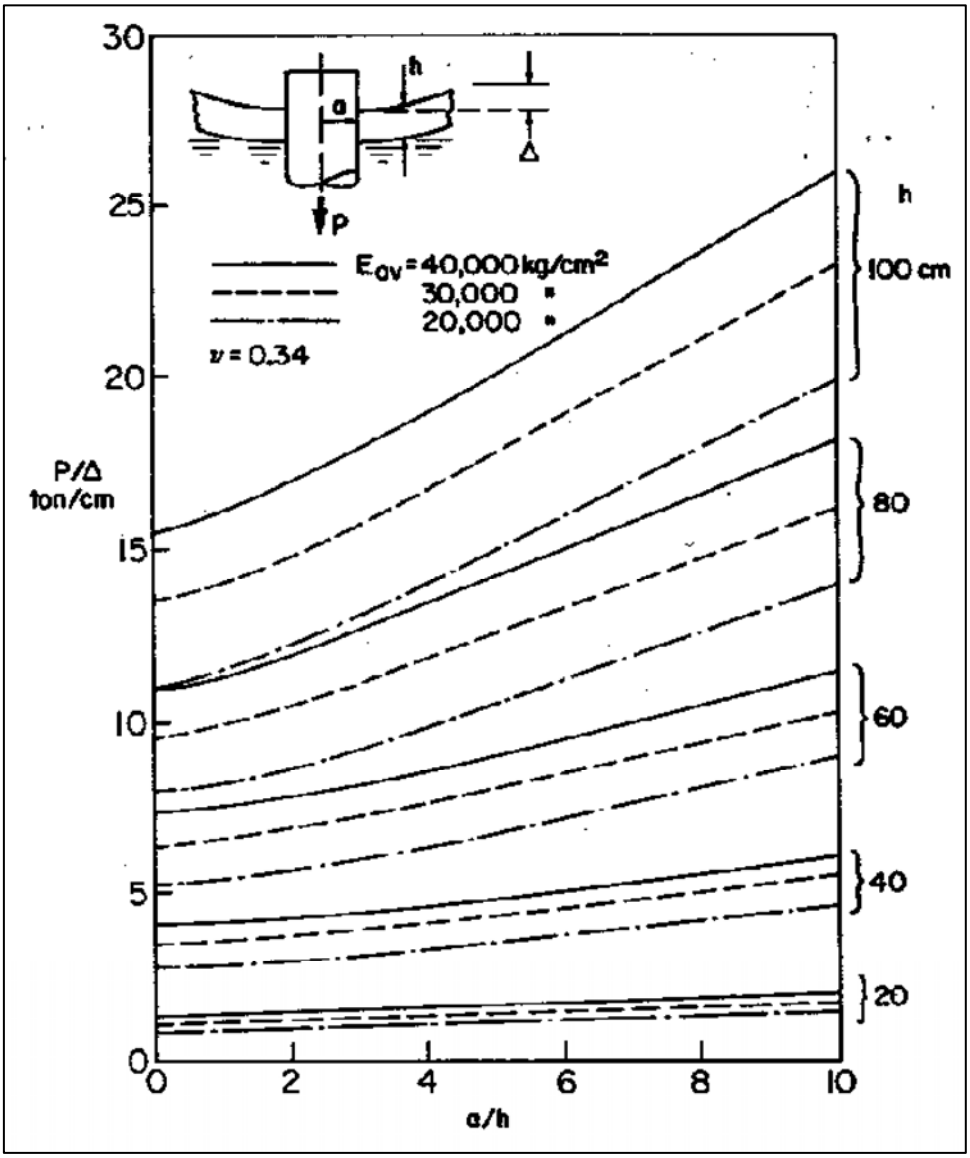

Figure VI-5-145. Vertical ice forces on a cylindrical pile (Kerr 1975)

$$P = f\ (a, h, E_{av}, \Delta, \nu) \qquad \text{(VI-5-320)}$$

where

P = uplift force in metric tons (tonnes)

a = pile radius (cm)

h = ice plate thickness (cm)

E_{av} = averaged Young's modulus for ice (kg/cm^2)

Δ = water level rise (cm) up to the thickness of the ice

v = Poisson's ratio

- Kerr's solution gives estimates of the maximum vertical load assuming the ice sheet does not fail in shear or bending before the maximum load on the pile is reached. For example, the maximum uplift force on a pile with radius a = 100 cm surrounded by a 40-cm-thick ice sheet having an average Young's modulus of 30,000 kg/cm^2 would be estimated from Figure VI-5-145 using a value of a/h = 2.5 giving $P_{max} = 3.7\,\Delta$. The total maximum force for a 5-cm water level rise would be

$$P_{\max} = 3.7\,\Delta = 3.7\,(5\,cm) = 18.5\,tonnes$$

- Kerr (1975) pointed out that the same analysis applied for falling water levels with only a sign change, thus Figure VI-5-145 can also be used decreasing water levels.

(b) Vertical walls.

- Uplift or downward forces per unit horizontal length caused by vertical movement of ice sheets frozen to vertical walls can be approximated using the following formula (PIANC 1992)

$$\frac{F_v}{b} = \rho_w\, g\, \Delta h\, L_c \qquad \text{(VI-5-321)}$$

where the characteristic length L_c is given as

$$L_c = \left[\frac{E\, h_i^3}{12\, \rho_w\, g\, (1 - v^2)}\right]^{1/4} \qquad \text{(VI-5-322)}$$

and

F_v = total vertical force acting on the wall

b = horizontal length of wall

Δh = change in water level

ρ_w = density of water

g = gravitational acceleration

E = modulus of elasticity of ice

h_i = ice thickness

v = Poisson's ratio (0.31-0.35)

- As previously mentioned, the modulus of elasticity for ice varies with brine volume from about 1,000 MPa for very salty water to about 6,000 MPa for freshwater ice. For freshwater ice, L_c is typically between 15 to 20 times the ice thickness with a reasonable rule-of-thumb being $L_c \approx 17\ h_i$.

(c) Sloping structures. The additional vertical load caused by the ride-up and piling of ice on sloping structures needs to be evaluated for the local conditions and specific type of structure. Ice piled up on the slope could initiate slumping of the armor layer on steeper slopes. During rising waters, individual armor stones or revetment units might be lifted out by adfreeze forces.

(4) Aspects of slope protection design.

(a) Much of our understanding of successful slope protection design in cold coastal regions stems from practical experience as documented in the technical literature. In general the design philosophy recognizes that little can be done to prevent ice contact with slope protection structures. Therefore, emphasis is placed on minimizing potential ice damage using a variety of techniques.

(b) Leidersdorf, Gadd, and McDougal (1990) reviewed the performance aspects of three types of slope protection used for coastal projects related to petroleum activities in the Beaufort Sea. For water depths less than 2 m, sacrificial beaches appeared to function well. In water depths ranging from 7 m to 15 m, gravel-filled geotextile bags were able to withstand the larger wave forces, but they were susceptible to ice damage and required regular maintenance. Linked concrete mat armor (Leidersdorf, Gadd, and McDougal 1988; Gadd and Leidersdorf 1990) withstood both wave and ice loads in depths up to about 14 m. Mats were recommended for projects with a lengthy service life so that high initial capital costs would be offset by lower maintenance expenses.

Wuebben (1995) reviewed the effects of ice on riprap structures constructed along ice-prone waterways. This paper provided a good summary of successful riprap revetment design and construction practices based on actual field experience. Numerous useful references documenting ice effects on riprap are included in Wuebben's paper.

The following rules-of-thumb for arctic slope protection were given in Chen and Leidersdorf (1988) and summarized in PIANC (1992).

- Cover layers and underlayers should be strong enough to withstand local penetration by thick ice sheets.

- Smooth slopes without protrusions will reduce loads and allow the ice to ride up more easily without plucking out individual armor elements. (However, wave runup will be greater.)

- Flexible cover layers consisting of graded riprap may help absorb impacts by smaller ice blocks during wave action without appreciable damage. Sand bags are effective for structures with intended short service lives.

- Mild structure slopes are essential because they reduce the risk of ice penetration into the slope. Maximum slope 15 deg is recommended in the zone of ice attack.

- Compound slopes with a nearly horizontal berm above the swl provide a platform for piled-up ice in regions which experience frequent ride-up of ice sheets.

- Maximum ice loads will not occur at the same time as maximum expected wave loads. Therefore, slope design can consider each load condition separately.

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Authors: Dr. Hans F.Burcharth, Department of Civil Engineering, Aalborg University, Aalbort, Denmark; and Dr. Steven A. Hughes, Coastal and Hydraulics Laboratory (CHL), U.S. Army Engineer Research and Development Center, Vicksburg, MS.

Reviewers:
H. Lee Butler, CHL (retired)
Dr, David R. Basco, Department of Civil Engineering, Old Dominion University, Norfolk, VA
Students Han Ligteringen, Delft University of Technology, The Netherlands; Jorgen Fredsøe and Mutly Sumer, Institute of Hydrodynamics and Hydraulic Engineering, Technical University of Denmark, Lyngby, Denmark.

VI-5-11. Symbols

α (alpha)

α	Angle a surface-piercing sloped plane forms with the horizontal [deg]
α	Angle of wave approach [deg]
α	Tangent of seaward armor slope

β (beta)

β	Angle of incidence of waves [deg]
β	Concave angle at vertical walls [radians]
β	Frequency parameter [dimensionless]

γ (gamma)

γ	Load factor [dimensionless]
γ_β	Factor for influence of angle of incidence β of the waves [dimensionless]
γ_b	Reduction factor for influence of a berm [dimensionless]
γ_h	Reduction factor for influence of shallow-water conditions where the wave height distribution deviates form the Rayleigh distribution [dimensionless]
γ_r	Reduction factor for influence of surface roughness [dimensionless]
γ_w	Specific weight of water or salt water [force/length^3]
_´	Average effective weight of soil from base to depth *B* under base level [force]

δ (delta)

δ	Logarithmic decrement
δ_0	Vertical shift in the wave crest and wave trough at the wall [length]

Δ (delta)

Δ	$(= \rho_s / \rho_w) - 1$

Δ	Water level rise up to the thickness of the ice [length]

$\Delta\tau_s$	Change in the average shear stress due to the submerged weight of the structure [force/length^2]
ε (epsilon)	
ε_i	Random wave phase angle of the *i*th incident wave component [deg]
$\dot{\varepsilon}_1, \dot{\varepsilon}_3$	Strain rates in principal stress directions 1 and 3
$\dot{\varepsilon}_{vol}$	Volume strain rate
η (eta)	
η	Sea surface elevation adjacent to a reflective structure [length]
η^2_{rms}	Root-mean-squared sea surface elevation [length^2]
θ (theta)	
θ	Angle of wave incidence [deg]
θ	Bottom slope [deg]
θ	Channel side wall slope [degrees]
θ	Wave phase angle (= $2\pi x/L$ - $2\pi t/T$) [radians]
θ_i	Reflection phase angle of the *i*th incident wave component [deg]
κ (kappa)	
κ	von Karman constant (= 0.4) [dimensionless]
λ (lamda)	
$\lambda_{1,2,3}$	Modification factors depending on the structure type [dimensionless]
μ (mu)	
μ	Dynamic friction coefficient corresponding to caisson displacement *S* [dimensionless]
μ	Friction coefficient for the base plate against the rubble stones [dimensionless]
μ	Structure slope friction factor [dimensionless]
ν (nu)	
ν	Kinematic viscosity [length^2/time]
ν	Poisson' ratio [dimensionless]
ν_*	Shear velocity
ξ (xi)	

ξ	Principal stress reduction factor
ξ_0	Surf similarity parameter for regular waves (Equation VI-5-1)
ξ_{0m}	Surf similarity parameter for irregular waves (Equation VI-5-2)
ξ_{0p}	Surf similarity parameter for irregular waves (Equation VI-5-2)
ξ_{eq}	Breaking wave surf similarity parameter

ρ (rho)

ρ	Bulk density [force/length^3]
ρ_a	Mass density of armor units [force/length^3]
ρ_c	Mass density of the structure [force/length^3]
ρ_i	Ice density [force/length^3]
ρ_s	Mass density of armor units [force/length^3]
ρ_w	Mass density of water (salt water = 1,025 kg/m^3 or 2.0 slugs/ft^3; fresh water = 1,000 kg/m^3 or 1.94 slugs/ft^3) [force-time^2/length^4]

σ (sigma)

σ	Normal stress on a section through a soil element [force/length^2]
σ	Spreading of short-crested waves
σ_1	Principal stress [force/length^2]
σ_c	Ice compressive failure strength in crushing [force/length^2]
σ_c	Standard deviation of the average non-dimensional cover armor depth
σ_e	Standard deviation of the average non-dimensional eroded armor depth
σ_f	Flexural strength of ice
σ_f'	Effective stress at failure [force/length^2]
σ_i	Angular wave frequency of the *i*th incident wave component [time^{-1}]
σ_n'	Normal stress on failure plane [force/length^2]
σ_S	Standard deviation of average damage

τ (tau)

τ	Shear stress on a section through a soil element [force/length^2]
τ_0	Shear stress acting on the bed [force/length^2]

Ψ (psi)

Ψ	Angle of dilation [degrees]
Ψ	Shields parameter

Ω (omega)

Ω	Dynamic load factor [dimensionless]

φ

φ	Angle of internal friction of the soil [degrees]
φ	Angle of repose of the armor [degrees]
φ	Strength factor [dimensionless]
φ'	Angle of friction in granular material [degrees]
φ'_{crit}	Critical angle of friction [degrees]
φ'_s	Effective secant angle of friction
φ'_t	Effective tangent angle of friction

A

a	Pile radius [length]
a_i	Amplitude of the *i*th incident wave component [length]
A	Area of structure slope [length^2]
A	Horizontal area of ice sheet [length^2]
A_c	Berm crest height [length]
A_e	Area of eroded armor layer [length^2]
A_n	Projected area of solid body normal to the flow direction [length^2]
A_s	Total area of steel intersecting the crack [length^2]
A_t	Area of initial cross section of structure [length^2]
A_z	Projected area of solid body in the horizontal plane [length^2]

B

b	Pile width [length]
b	Structure horizontal width or diameter [length]

B	Berm width [length]
B	Diameter of the vertical breakwater circular head [length]
B	Function of Reynolds number (= 8.5 for fully rough, turbulent flow)
B	Horizontal width of the barrier [length]
B	Relative breakage
B	Width of footing [length]
B	Width of structure crest [length]
C	
c	Infiltration factor [dimensionless]
c	Shear strength of soil [force/length^2]
c_u	Undrained shear strength [force/length^2]
c'	Cohesion intercept
C	Damage parameter for structure armor layer [dimensionless]
C	Dolos fluke length [length]
C_0	Zero-damage cover layer thickness [length]
C_D	Drag hydrodynamic force coefficient [force/length]
C_L	Empirical lift coefficient
C_M	Inertia or mass hydrodynamic force coefficient [force/length]
C_r	Bulk reflection coefficient [dimensionless]
C_{ri}	Reflection coefficient of the *i*th incident wave component [dimensionless]
C_s	Stability coefficient for incipient motion [dimensionless]
C_{sf}	Coefficient of skin friction between wind and ice or water and ice [dimensionless]
C_t	Wave transmission coefficient [dimensionless]
C_{to}	Wave transmission coefficient for overtopping [dimensionless]
C_{tp}	Wave transmission coefficient for wave penetration [dimension]
C_u	Uncertainty factor [dimensionless]
C_U	Laboratory derived slamming coefficient [dimensionless]
C_V	Coefficient of consolidation

D

d	Grain diameter [length]
d_B	Berm horizontal surface above the still-water line [length]
d_c	Depth of armor cover [length]
d_c	Elevation of the lower edge of the sloping face relative to the SWL [length]
d_e	depth of sheet-pile penetration below the seabed [length]
d_e	Depth of eroded armor layer [length]
d_i	Depth at the toe of the sloping structure [length]
d_s	Water depth at the structure toe [length]
D	Cylindrical pile diameter [length]
D	Damping ratio [dimensionless]
D	Minimum depth of footing below soil surface [length]
D	Pipe diameter [length]
D	Sphere diameter [length]
D_e	Equivalent pile diameter [length]
D_H	Distance between centroids of two adjacent units on the same horizontal row [length]
D_n	Cube length [length]
D_{n50}	Median of nominal diameter of rocks for design conditions [length]
D_r	Relative density of soils [percent]
D_{swl}	Vertical distance from SWL to location of stressed dolos [length]
D_U	Distance between the centroids of units upslope in the plane of the structure slope [length]

E

e	In-place void ratio [dimensionless]
e_{cr}	Critical embedment [length]
e_{max}	Void ratio of soil in loosest condition [dimensionless]
e_{min}	Void ratio of soil in most dense condition [dimensionless]
E	Damage parameter for structure armor layer [dimensionless]

E	Modulus of elasticity of ice
E	Young's modulus
E_{av}	Averaged Young's modulus for ice [force/length^2]
E_d	Dissipated wave energy in one wavelength per unit crest width [length-force/length^2]
E_i	Incident wave energy in one wavelength per unit crest width [length-force/length^2]
E_r	Reflected wave energy in one wavelength per unit crest width [length-force/length^2]
E_t	Transmitted wave energy in one wavelength per unit crest width [length-force/length^2]
$\boldsymbol{F}$	
f_c	Concrete compressive strength [force/length^2]
f_c	Height of wall not protected by the armor layer [length]
f_{ct}	Concrete splitting tensile strength [force/length^2]
f_D	Drag force per unit length of pile [force/length]
f_i	Inertial force per unit length of pile [force/length]
f_i	Reduction factor [dimensionless]
f_T	Concrete static tensile strength [force/length]
f_y	Yield strength of the steel
F	Safety factor [dimensionless]
F_0	Significant force per unit width for a vertical wall [force/length]
F_B	Buoyancy force [force]
F_c	Horizontal crushing force [force]
F_D	Drag force [force]
F_G	Gravitational force [force]
F_G	Reduced weight of the vertical structure due to buoyancy [force]
F_H	Wave induced horizontal force [force]
F_I	Inertia force [force]
F_L	Lift force [force]
F_L	Time-varying transverse (lift) force
F_{Lm}	Maximum transverse force [force]

F_{m0}	Significant force per unit width of barrier [force/length]
F_r	Flow Froude number [dimensionless]
F_U	Wave induced uplift force [force]
F_v	Total vertical force acting on the wall [force]
F_W	Irregular wave loading [force]
G	
g	Gravitational acceleration [length/time^2]
G	Berm width [length]
G	Factor dependent on the armor layer gradation [dimensionless]
G	Shear modulus
H	
h	Ice plate thickness [length]
h	Pre-scour water depth at the vertical wall [length]
h	Water depth [length]
h_b	Water depth at a distance of $5H_s$ seaward of the breakwater front wall [length]
h_b	Water depth at top of toe berm [length]
h_c	Equilibrium height of the structure [length]
h_i	Thickness of ice sheet [length]
h_s	Water depth in front of structure [length]
h'	Height of wall protected by the armor layer [length]
h'	Submerged height of the wall from the toe to the still water line [length]
h'_c	Initial height of structure over seabed level [length]
Δh	Change in water level [length]
H	Characteristic wave height [length]
H	Drainage distance [length]
H_0	Deepwater wave height [length]
H_b	Breaking wave height [length]
H_c	Wave height in the corner [length]
H_i	Incident wave height [length]

H_I	Incident wave height [length]
H_{m0}	Zeroth-moment wave height [length]
H_S	Significant wave height [length]
H_{sr}	Significant reflected wave height [length]
H_w	Wave or surge height at the wall [length]
I	
i	Hydraulic gradient [length/length]
I	Indentation coefficient [dimensionless]
I_p	Plasticity index
K	
k	Contact coefficient [dimensionless]
k	Permeability coefficient
k	Wave number (= $2\pi/L = 2\pi/CT$) [length^{-1}]
k_i	Wave number of the *i*th incident wave component [length^{-1}]
k_M	Moment contribution factor
k_p	Wave number associated with the spectral peak by linear wave theory [length^{-1}]
k_p	Wave number associated with the spectral peak period T_p [length^{-1}]
k_s	Boundary or bed roughness
k_T	Torque contribution factor [dimensionless]
k_x	Stiffness coefficient
k_φ	Stiffness coefficient
k_Δ	Layer coefficient (Table VI-5-51)
K	Bulk modulus
K	Coefficient of lateral stress [dimensionless]
K	Factor to account for blankets plaved on sloping channel side walls [dimensionless]
K_1	Pile shape factor [dimensionless]
K_2	Pile orientation factor [dimensionless]
KC	Keulegan-Carpenter number [dimensionless]
K_D	Stability coefficient

K_o	Coefficient of lateral stress at rest [dimensionless]
L	
l_e	Upslope eroded length [length]
L	Damage parameter for structure armor layer [dimensionless]
L	Length of footing [length]
L	Local wave length [length]
L_0	Deepwater wave length (= $gT^2/2\pi$) [length]
L_{0m}	Deepwater wave length corresponding to mean wave period [length]
L_{0p}	Deepwater wave length corresponding to the peak of the wave spectrum [length]
L_p	Local wavelength associated with the peak spectral period T_p [length]
M	
m	Plan shape coefficient [dimensionless]
m	Total mass [force]
m_0	Area beneath the measured force spectrum [length^2]
M	Armor unit mass [force]
M	Constrained modulus
M_{50}	Medium mass of rocks; mass of Core-Loc armor unit (= ρ_s $(D_{n50})^3$) [force]
M_{cr}	Critical strength of concrete in moment
M_d	Overturning moment per unit horizontal length about the toe of the wall due to the dynamic pressure [length-force/length]
M_{FG}	Stabilizing moment around the heel by buoyancy-reduced weight of the caisson [length-force]
M_{FH}	Antistabilizing moment by wave induced horizontal force [length-force]
M_{FU}	Antistabilizing moment by wave induced uplift force [length-force]
M_{max}	Maximum wave-load-induced moment around the center of gravity [length-force]
M_s	Hydrostatic overturning moment per unit width [length-force/length]
M_s	Stabilizing moment due to friction and cohesion [length-force]

N

n	Model scale factor [dimensionless]
n	Porosity
$\underline{n}_z$	Normal unit velocity in the positive z-direction [length/time]
N_{0w}	Number of overtopping waves
N_a	Total number of armor layer units or number of rocks in the mound
N_f	Number of cycles to failure
N_{od}	Number of units displaced out of the armor layer
N_{or}	Number of rocking units
N_S	Stability parameter [dimensionless]
N_w	Number of incoming waves
N_z	Number of waves
N^*_S	Spectral stability number [dimensionless]

P

p	Porosity of the armor layer [dimensionless]
p_a	Atmospheric pressure [force/length^2]
p_s	Pressure on solid body surface due to moving fluid [force/length^2]
$p_{1,2,3}$	Wave pressure at the SWL corresponding to wave crest, at the base, at the SWL, corresponding to wave trough [force/length^2]
p'	Mean effective stress [force/length^2]
p'_o	Vertical effective overburden pressure [force/length^2]
P	Notational permeability parameter (Figure VI-5-11)
P	Uplift force in metric tons
P_{ow}	Probability of overtopping per incoming wave
P_s	Hydrostatic pressure [force/length^2]

Q

q	Average overtopping discharge per unit length of structure [length^3/time/length]
q'	Effective overburden pressure [force/length^2]
Q	Dimensionless average discharge per meter (Equations VI-5-20 and VI-5-21)

Q_n	Nominal load [force]
R	
r	Dolos waist ratio [dimensionless]
r	Thickness of armor cover or under layer [length]
R_a	Maximum vertical runup height [length]
R_c	Crest freeboard (Figure VI-5-13) [length]
R_d	Force per unit horizontal length of wall [force/length]
R_d	Minimum rundown or water-surface elevation measured vertically from the still-water level [length]
R_e	Reynolds number [dimensionless]
R_h	Distance to the center of the section [length]
R_n	Nominal strength
R_s	Hydrostatic force per unit horizontal width of wall [force/length^2/length]
R_u	Maximum runup or water-surface elevation measured vertically from the still-water level [length]
$R_{ui\%}$	Runup level exceeded by *i* percent of the incident waves (Equation VI-5-3) [length]
R_{us}	Significant runup level [length]
S	
s_0	Deepwater wave steepness ($=H_0/L_0$) [dimensionless]
s_{0m}	Deepwater mean wave steepness ($=H_s/L_{0m}$)
s_{0p}	Deepwater wave steepness corresponding to the peak of the wave spectrum [dimensionless]
s_m	Wave steepness ($= H_s / L_{0m}$)
s_p	Local wave steepness [dimensionless]
s_t	Settlement (decrease in layer thickness) at time *t* [length]
s_∞	Final settlement reached when the soil skeleton is fully carrying the load [length]
S	Caisson displacement [length]
S	Horizontal seismic inertia force
S	Relative eroded area or damage parameter for structure armor layer [dimensionless]
S_f	Safety factor at allow for debris impacts or other unknowns [dimensionless]

S_m	Maximum scour depth [length]
S_M	Section moduli for flexure
S_t	Cohesive soil sensitivity (ratio between the undrained shear strength of a specimen in undisturbed and in remoulded states)
S_T	Section moduli for torsion
T	
t	Time at end of storm n
t_n	Time at start of storm n
T	Wave period [time]
T_{0m}	Wave period associated with the spectral peak in deep water [time]
T_{0p}	Wave period associated with the spectral peak in deep water [time]
T_{cr}	Critical strength of concrete in torque
T_m	Average or mean wave period [time]
T_p	Wave period corresponding to the peak of the wave spectrum [time]
T_s	Strength contribution from the torsional steel reinforcement
U	
u	Horizontal component of the wave orbital velocity [length/time]
u	Magnitude of flow velocity [length/time]
u	Pore pressure [force/length^2]
u_i	Velocity of ice [length/time]
u_p	Pore water pressure [force/length^2]
u_s	Water pressure along the surface of the slope [force/length^2]
u^2_{rms}	Root-mean-squared horizontal wave velocity [length^2]
U	Current magnitude [length/time]
U	Degree of consolidation [dimensionless]
U_c	Critical depth-averaged flow velocity [length/time]
U_m	Maximum wave orbital velocity at the bed [length/time]

V

v	Bulk flow velocity [length/time]
V	Overtopping volume per wave per unit width [length^3/length]
V	Total volume [length^3]
V_p	Volume of voids [length^3]
V_s	Volume of solids [length^3]

W

w	Barrier penetration depth [length]
w	Vertical component of flow velocity at level of object [length/time]
w_a	Specific weight of armor material [force/length^3]
W	Stone or armor weight [force]
W	Total weight of the slice including surface load [force]
W	Width of scour apron [length]
W_T	Total weight of riprap [force]

X

x	Horizontal coordinate with positive toward the structure and x = 0 located at the structure toe [length]
x	Strain rate function

Y

y	Elevation above the bed [length]

Z

z	Vertical coordinate with z = 0 at the SWL and z = -h at bottom [length]
Z	Maximum vertical ice ride-up distance [length]