PAPER NUMBER: H43B-0499 MORE EFFICIENT DERIVATIVE-BASED WATERSHED MODEL CALIBRATION

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1. BACKGROUND

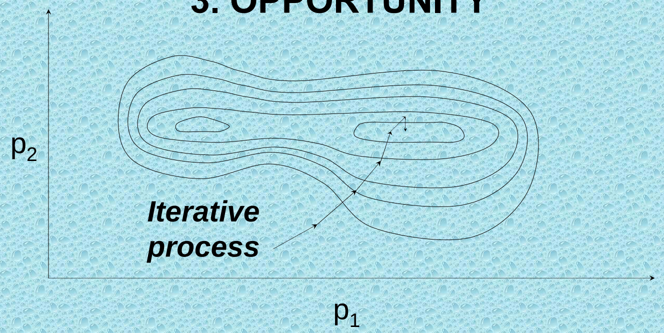
Model independent enhancements and adaptations to the Levenburg-Marquardt (LM) parameter estimation method have recently been shown to be useful in the watershed model calibration context (e.g., Skahill and Doherty 2006; Doherty and Skahill 2006).

2. LM UPDATE RULE

$$\Delta \mathbf{p} = (\mathbf{H} + \lambda \, diag[\mathbf{H}])^{-1} \nabla f$$

p, f, **H**, and λ represent the vector of m adjustable model parameters, the weighted least squares objective function, the Hessian of f, and the LM update parameter.

3. OPPORTUNITY



$$\mathbf{J} = \left[\frac{\partial f_i}{\partial p_j}\right] \approx \frac{f_i(p + \delta e_j) - f_i(p)}{\delta}$$

Numerical Differentiation (FD)

In recent works such as the two referenced above, the Jacobian, **J**, at each optimization iteration was approximated using software that requires between m and $2m$ forward model calls (simplest remedy, but ***expensive***).

4. OBJECTIVE

Implement a secant version of the LM method into our model independent solver and subsequently begin to examine its efficiency for watershed model calibration.

5. METHODS

Employed Broyden's Update (secant approx. to the derivative along the search direction)

- Allowed for cyclic updating (wherein columns are periodically updated via FD)
- Also included ability to recalculate the Jacobian using FD when deemed necessary

Calibrated 8 parameter HSPF, 6 parameter HEC-HMS, and 20 parameter HSPF models using the secant version of LM. Compared results with conventional LM application approach (e.g., Doherty, 2005).

6. RESULTS

LM Method Single Inversion Runs for 8 parameter HSPF model

	Conventional	Secant Version							
		# Columns used with Cyclic Updating							
		Full Jacobian not recalculated							
		0	1	2	3	4	5	6	7
Total Model Calls	159	58	95	92	124	122	156	101	183
Final Objective Function Value	23.60	24.86	23.68	23.58	23.57	23.57	23.99	23.58	23.58

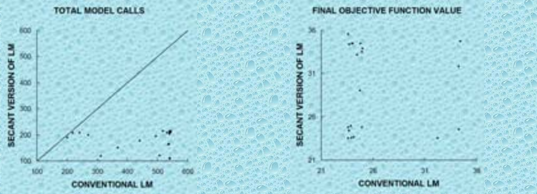
LM Method Single Inversion Runs for 6 parameter HEC-HMS model

	Conventional	Secant Version					
		# Columns used with Cyclic Updating					
		Full Jacobian not recalculated					
		0	1	2	3	4	5
Total Model Calls	218	63	81	89	141	112	117
Final Objective Function Value	502.76	518.21	502.78	502.73	531.50	502.73	506.76

LM Method Single Inversion Runs for 20 parameter HSPF model

	Conventional	Secant Version					
		# Columns used with Cyclic Updating					
		Full Jacobian not recalculated					
		0	1	2	3	5	10
Total Model Calls	548	39	46	107	203	268	336
Final Objective Function Value	48.66	114.18	93.68	51.15	49.25	48.88	48.90

Simple multistart – 20 Inversion Runs for 8 parameter HSPF model
The Secant Version of LM used 2 columns for Cyclic Updating



7. CONCLUSIONS

- Implemented a secant version of the LM method (not new) *and applied it in the watershed model calibration context (new)*
- Across the 3 case study applications, the secant version of LM reduced the total number of model calls by as much as ~60 percent relative to conventional updating of the Jacobian, with little or no loss in objective function improvement
- Future research to focus on cyclic updating and column selection, and improving efficiency of LM-based global optimization (Skahill and Baggett, 2007) and regularization adaptations through use of a secant approx. to the derivative

8. REFERENCES

Skahill, B., and Doherty, J. 2006. Efficient accommodation of local minima in watershed model calibration. Journal of Hydrology, 329, 122-139.
Doherty, J., and Skahill, B. 2006. An Advanced Regularization Methodology for Use in Watershed Model Calibration. Journal of Hydrology, 327, 564– 577.
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