Reliability Analysis Procedures for Infrastructure Facilities

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Outline

- Causes of Uncertainty
- Load Model
- Resistance Model
- Reliability Analysis
- Target Reliability
- Design Codes

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Infrastructure Facilities

- Buildings (offices, residential structures, hospitals, ...)
- Transportation facilities (bridges, roads, retaining walls, ...)
- Industrial structures
- Pipelines
- Hydraulic structures (dams, levees, ...)

Need for Protection

- Infrastructure facilities need protection
- Natural causes (extreme events: hurricanes, floods, tornados, major storms, snow, earthquakes)
- Man-made causes (poor maintenance, vehicle collisions, vessel collisions, human errors, terrorist attacks)
- Limited available resources (the needs must be prioritized)

Uncertainties in Load and Resistance (Load Carrying Capacity)

Load and resistance parameters have to be treated as random variables

- Occurrence probability (return period)
- Magnitude (mean values, coefficient of variation)
- Time variation (stochastic processes)

Load and Resistance are Random Variables

- Dead load, live load, dynamic load
- Natural loads – temperature, water pressure, earth pressure, wind, snow, earthquake, ice
- Material properties – concrete, steel, wood, plastics, composites
- Dimensions
- Man-made causes – collisions (vehicle, vessel), fire, poor maintenance, human errors, gas explosion, terrorist acts
- Load effects – analytical models, approximations
- Load combinations

Consequences of Uncertainties

- Deterministic analysis and design is insufficient
- Probability of failure is never zero
- Design codes must include a rational safety reserve (too safe – too costly, otherwise – too many failures)
- Reliability is an efficient measure of the structural performance

Reliability and Risk (Probability of Failure)

- Reliability = probability that the structure will perform its function during the predetermined lifetime
- Risk (or probability of failure) = probability that the structure will fail to perform its function during the predetermined lifetime

Current approach to risk is not rational

Risk perception is different depending on the cause:

- Earthquake
- Flood
- Corrosion
- Poor maintenance
- Collision
- Human error
- Terrorist act

Basic Needs

- How to measure risk?
- What is the actual risk?
- What is the optimum (acceptable) risk?
- How to implement the optimum risk in practice?
- How to design infrastructure systems?
- How to up-grade existing infrastructure systems?
- How to manage the infrastructure assets?

How to Measure Risk?

- Identify the load and resistance parameters

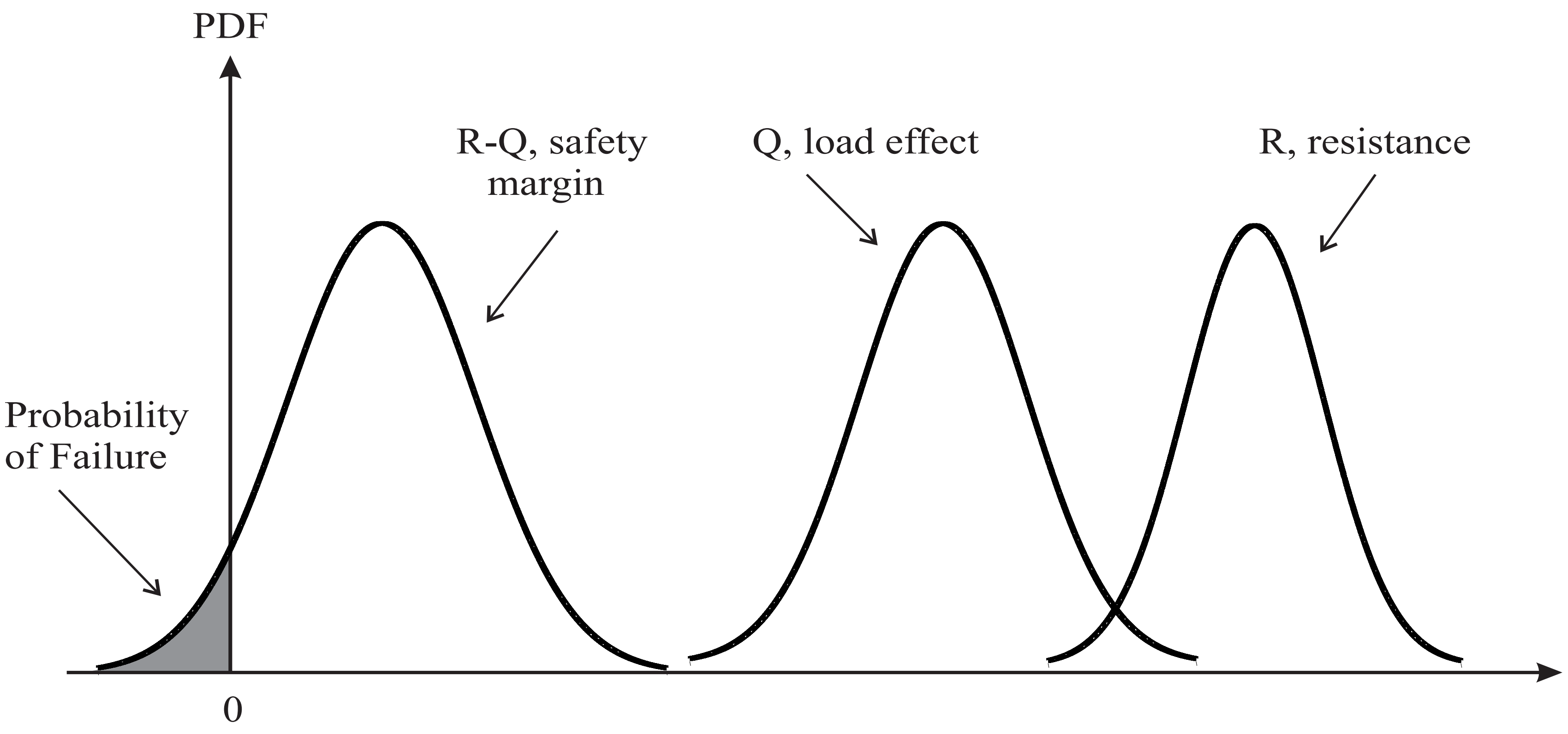
  $(X_1, \ldots, X_n)$

- Formulate the limit state function, $g(X_1, \ldots, X_n)$, such that $g < 0$ for failure, and $g \geq 0$ for safe performance

- Calculate the risk (probability of failure, P_F,

  $P_F = \text{Prob}(g < 0)$

Fundamental Case
Safety Margin, g = R – Q
what is the probability g < 0?



Fundamental case

The limit state function, g = R - Q, the probability of failure, P_F, can be derived considering the PDF's of R and Q

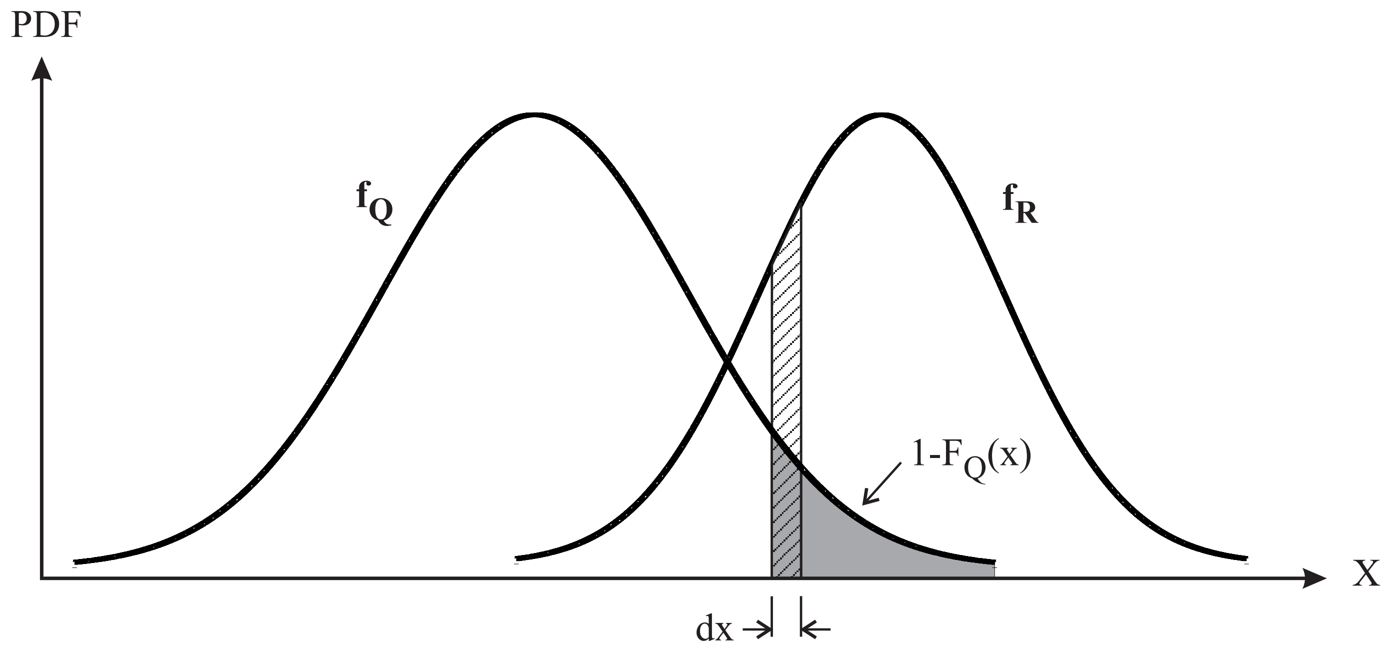


Figure 5-9 PDFs of load (Q) and resistance (R).

Fundamental case

The structure fails when the load exceeds the resistance, then the probability of failure is equal to the probability of Q>R, the following equations result

$$P_f = \sum P(R = r_i \cap Q > r_i) = \sum P(Q > R \mid R = r_i) P(R = r_i)$$

$$P_f = \int_{-\infty}^{+\infty} \left(1 - F_Q(r_i)\right) f_R(r_i) dr_i = 1 - \int_{-\infty}^{+\infty} F_Q(r_i) f_R(r_i) dr_i$$

$$P_f = \sum P(Q = q_i \cap R < q_i) = \sum P(R < Q \mid Q = q_i) P(Q = q_i)$$

$$P_f = \int_{-\infty}^{+\infty} F_R(q_i) f_Q(q_i) dq_i$$

Too difficult to use, therefore, other procedures are used

Fundamental case

- Space of State Variables

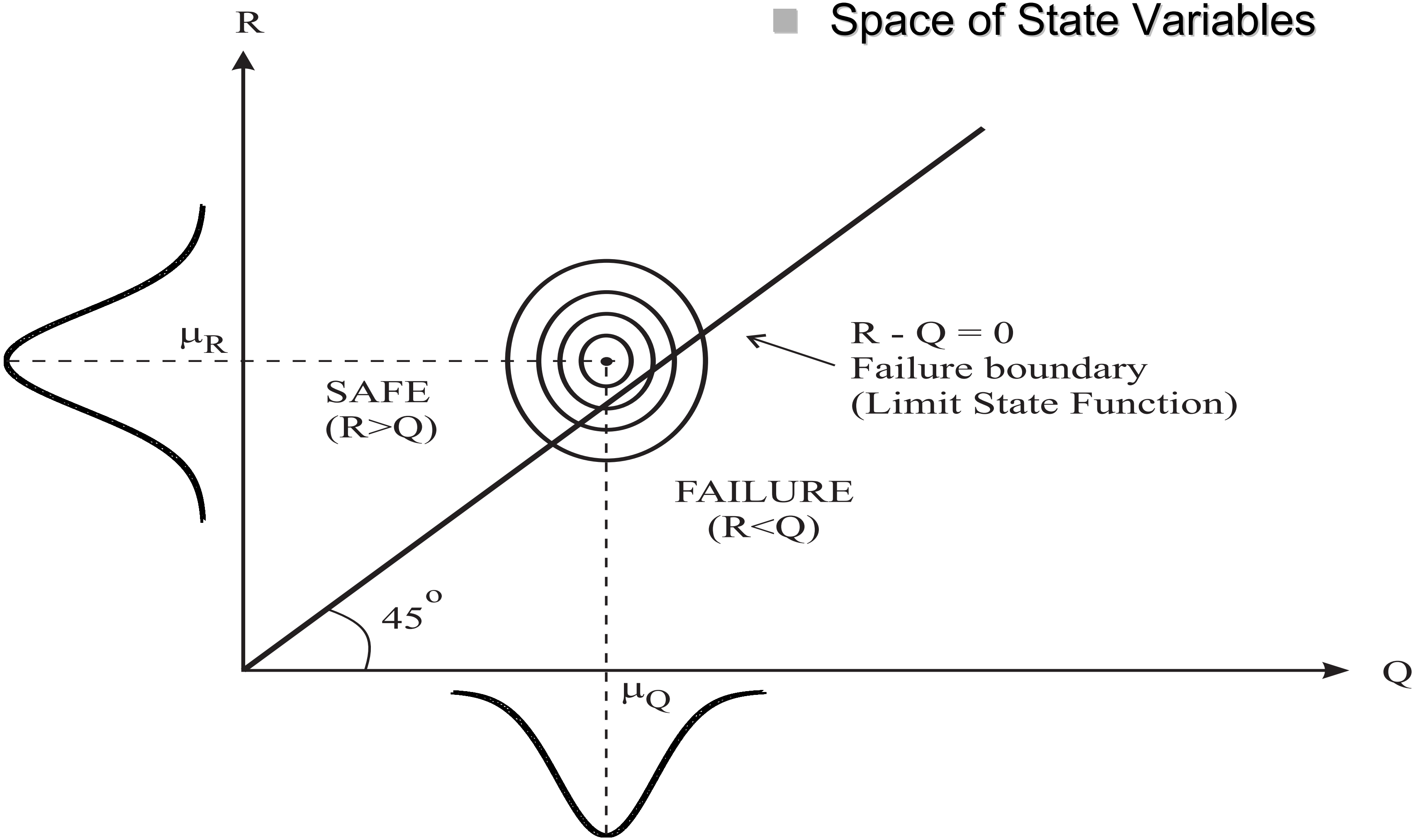


Figure 5-10 Safe domain and failure domain in a two-dimensional state space.

Fundamental case

- Space of State Variables

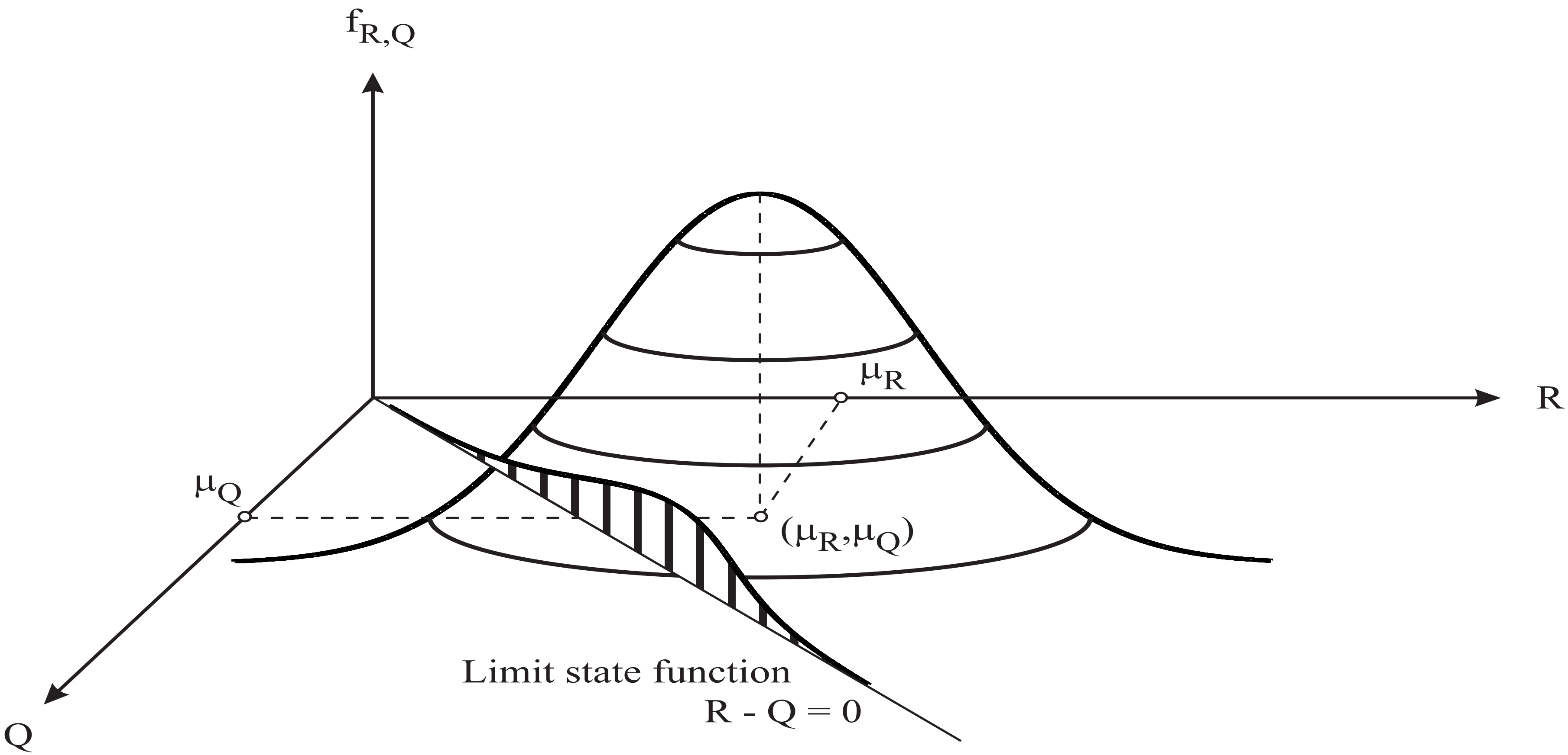


Figure 5-11 Three-dimensional sketch of a possible joint density function f_{RQ}.

Reliability Index, β

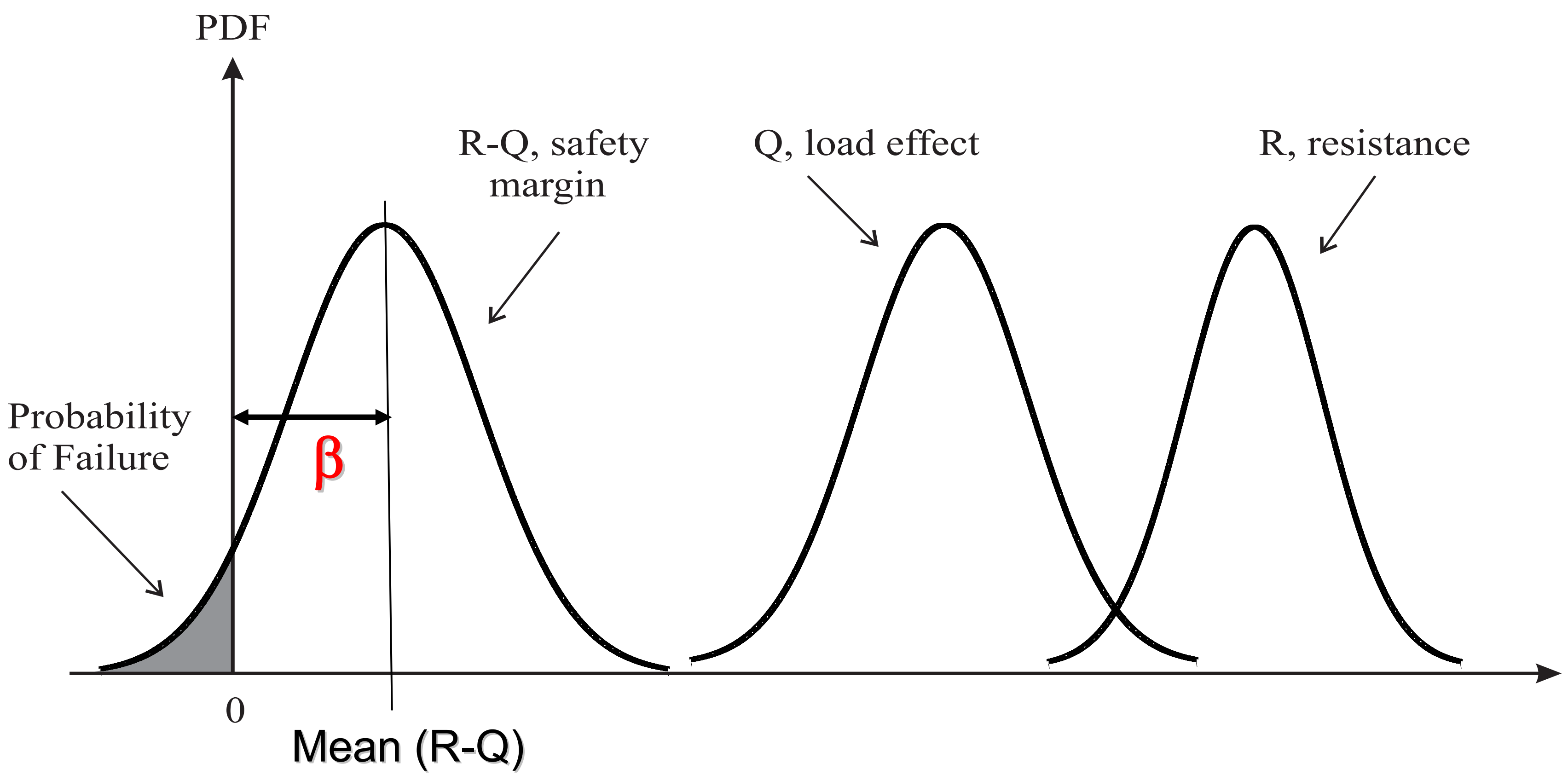


Figure 5-8 PDFs of load, resistance, and safety margin.

Reliability Index, β

For a linear limit state function, $g = R - Q = 0$, and R and Q both being normal random variables

$$\beta = \frac{(\mu_R - \mu_Q)}{\sqrt{\sigma_R^2 + \sigma_Q^2}}$$

μ_R = mean resistance

μ_Q = mean load

σ_R = standard deviation of resistance

σ_Q = standard deviation of load

Reliability Index and Probability of Failure

Reliability index, β

$$\beta = -\Phi^{-1}(P_F)$$

$$P_F = \Phi(-\beta)$$

Reliability index and probability of failure

P_F	β
10^{-1}	1.28
10^{-2}	2.33
10^{-3}	3.09
10^{-4}	3.71
10^{-5}	4.26
10^{-6}	4.75
10^{-7}	5.19
10^{-8}	5.62
10^{-9}	5.99

Reliability Analysis Procedures

- Closed-form equations – accurate results only for special cases
- First Order Reliability Methods (FORM), reliability index is calculated by iterations
- Second Order Reliability Methods (SORM), and other advanced procedures
- Monte Carlo method - values of random variables are simulated (generated by computer), accuracy depends on the number of computer simulations

Reliability Index - Closed-Form Solution

- Let's consider a linear limit state function

$$g\,(X_1, X_2, \ldots, X_n) = a_0 + a_1 X_1 + a_2 X_2 + \ldots + a_n X_n$$

- X_i = uncorrelated random variables, with unknown types of distribution, but with known mean values and standard deviations

$$\beta = \frac{a_0 + \sum_{i=1}^{n} a_i \mu_{X_i}}{\sqrt{\sum_{i=1}^{n} (a_i \sigma_{X_i})^2}}$$

Reliability Index for a Non-linear Limit State Function

- Let's consider a non-linear limit state function

  $g\ (X_1, \ldots, X_n)$

- X_i = uncorrelated random variables, with unknown types of distribution, but with known mean values and standard deviations

- Use a Taylor series expansion

$$g(X_1, X_2, \ldots, X_n) \approx g\left(x_1^*, \ldots, x_n^*\right) + \sum_{i=1}^{n} \left(X_i - x_i^*\right) \frac{\partial g}{\partial X_i}$$

where the derivatives are calculated at $(X_1^*, \ldots, X_n^*)$

Reliability Index – Nonlinear Limit State Function

$$\beta = \frac{a_0 + \sum_{i=1}^{n} a_i \mu_{X_i}}{\sqrt{\sum_{i=1}^{n} (a_i \sigma_{X_i})^2}}$$

where $a_i = \frac{\partial g}{\partial X_i}$ calculated at $(X_1^*, \ldots, X_n^*)$.

But how to determine $(X_1^*, \ldots, X_n^*)$?

It is called the design point.

Iterative Solutions for β

- Hasofer-Lind Reliability Index
  - Non-linear limit state function
  - Input data for each variable: mean and standard deviation

- Rackwitz-Fiessler procedure
  - Input data for each variable: cumulative distribution function (CDF)

Monte Carlo Simulations

- Given limit state function, g (X_1, …, X_n) and cumulative distribution function for each random variable X_1, …, X_n
- Generate values for variables (X_1, …, X_n) using computer random number generator
- For each set of generated values of (X_1, …, X_n) calculate value of g (X_1, …, X_n), and save it
- Repeat this N number of times (N is usually very large, e.g. 1 million)
- Calculate probability of failure and/or reliability index
  - Count the number of negative values of g, NEG, then P_F = NEG/N
  - Plot the cumulative distribution function (CDF) of g on the normal probability paper and either read the resulting value pf P_F and β directly from the graph, or extrapolate the lower tail of CDF, and read from the graph.

Typical Values of β

- Structural components (beams, slabs, columns), β = 3-4
- Connections:
  - welded β = 3-4
  - bolted β = 5-7
- Structural systems (building frames, girder bridges) β = 6-8

Selection Criteria for the Target Reliability Index

- Consequences of failure
- Marginal cost of reliability (cost to increase or decrease the reliability by a unit)
- Reliability of structures designed using the current (old) code
- Performance of structures designed using the current (old) code

Limit States

- Strength limit state
- Service limit state
- Fatigue and fracture limit state
- Extreme event limit state

Target Reliability Index – major considerations

- Primary and secondary components
- Multiple and single load paths (redundancy)
- Element and system reliability
- New design and existing structure
- Ductile and brittle materials and components
- Important and ordinary structures

Types of Components

- Primary component – its failure causes failure of other components (or progressive or total collapse)
- Secondary component – its failure does not affect performance of other components

New Design vs.
Existing Structure

- For a new design, reliability can be increased with little extra cost
- For an existing structure any strengthening can be prohibitively expensive
- Current practice accepts lower reliability levels for existing structures

System vs. Component

- Structures are systems made of components
- Series system (weakest link)
- Parallel systems
- Failure of a component may not mean failure of the system
- Ductile and brittle components
- Correlation between components

Examples of the Target Reliability Indices for Bridge Components

- $\beta_T = 3.5$
  - Primary component
  - Multiple load path
- $\beta_T = 5.0$
  - Primary component
  - Single load path
- $\beta_T = 2.0$
  - Secondary component

Examples of the Target Reliability Indices for Bridges

- For steel, reinforced concrete, prestressed concrete girders,
  - $\beta_T = 3.5$
- For sawn wood bridge components,
  - $\beta_T = 2.0$
- For girder bridge as a system,
  - $\beta_T = 5.5\text{-}6.5$

Target Reliability Indices for Bridge Components

- Case 1 with $\beta_T = 3.5$
  - Primary component
  - Multiple load path
- Case 2 with $\beta_T = 5.0$
  - Primary component
  - Single load path
- Case 3 with $\beta_T = 2.0$
  - Secondary component

Reliability of Connections

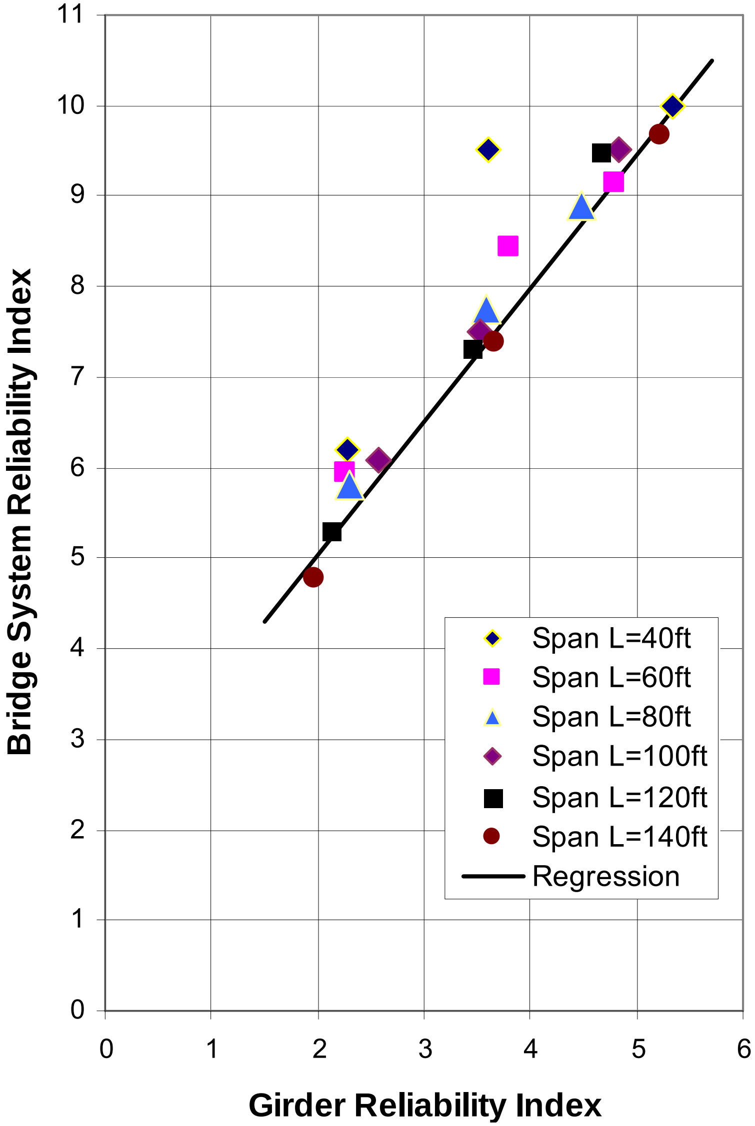
- For a bolted connection, the reliability can be increased with negligible extra cost (extra bolts)
- For a steel component, the increase of reliability is much more costly (heavier section)
- Target reliability index for bolts is β_T = 5-6, while for beams, β_T = 3-4

System Reliability vs. Girder Reliability

for Different Spans

Girder spacing,

S = 6ft



Conclusions

- Load and resistance parameters are random variables, therefore, reliability can serve as an efficient measure of structural performance
- Reliability methods are available for the analysis of components and complex systems
- Target reliability indices depend on consequences of failure or costs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