Coastal Engineering Technical Note

EXTREMAL SIGNIFICANT WAVE HEIGHT DISTRIBUTIONS COMPUTER PROGRAM: WAVDIST (MACE-17)

PROGRAM PURPOSE: The program WAVDIST estimates the parameters of three commonly applied probability distributions used in the prediction of extreme wave conditions. The program, in its present form, assumes that all the wave height properties of a storm sea can be represented by the significant or spectrally based (zero moment) wave height. Extremes are typically defined as those events in which the peak conditions of the storm exceed a subjectively set threshold. It is suggested that this threshold be set to yield an annual average number of extreme events of one or more. A discussion on the confidence of predicted extreme values is given in CETN-I-5.

PROGRAM CAPABILITY: The program WAVDIST computes the parameters for each distribution by a least squares fit for up to 200 extreme wave height values entered via the keyboard. The parameters, mean, and variance of each distribution are printed. The output also includes several goodness of fit criterion for each distribution fitted: the non-linear correlation, the sum of the squared residuals, the standard error of the estimate, and the mean square deviation. The average number of extreme events per year (also termed the Poisson lambda parameter) is also included in the printout. A return period table and an optional residual table may also be printed for each distribution. Alternate versions of WAVDIST that plot the data and the fit of the distributions on an HP 7475 pen plotter or on Tektronix terminals and/or a version that estimates the parameters by the method of moments are available.

BACKGROUND: Three of the most frequently used probability distributions for describing extreme wave statistics are the three fitted in this program:

(1) Extremal Type I (also termed Gumbel and Fisher-Tippett Type I)

(2) Weibull

(3) Log-Extremal

Table 1 summarizes the properties of each distribution. The cumulative form of these distributions is for non-exceedance of a random variable significant (or zero moment) wave height denoted by H_s. A specific significant wave height is denoted by h_s.

Table 1

Definition of Distributions

Distribution	Domain	Cumulative Probability Function: $F(h_s) = P_r(H_s \leq h_s)$	Probability Density Function: $f(h_s)$	Mean	Variance
Extremal Type I	$-\infty < h_s < \infty$ $-\infty < \varepsilon < \infty$ $0 < \phi < \infty$	$F(h_s) = e^{-e^{-\left(\frac{h_s-\varepsilon}{\phi}\right)}}$	$f(h_s) = \dfrac{e^{-e^{-\left(\frac{h_s-\varepsilon}{\phi}\right)}} \, e^{-\left(\frac{h_s-\varepsilon}{\phi}\right)}}{\phi}$	$\varepsilon + \gamma\phi$	$\frac{\pi^2}{6}\phi^2$
Weibull	$0 < h_s < \infty$ $0 < \beta < \infty$ $0 < \alpha < \infty$	$F(h_s) = 1 - e^{-\left(\frac{h_s}{\beta}\right)^{\alpha}}$	$f(h_s) = \dfrac{\alpha h_s^{\alpha-1}}{\beta^{\alpha}} e^{-\left(\frac{h_s}{\beta}\right)^{\alpha}}$	$\beta\Gamma\left(1 + \frac{1}{\alpha}\right)$	$\beta^2\left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \Gamma^2\left(1 + \frac{1}{\alpha}\right)\right]$
Log - Extremal	$0 < h_s < \infty$ $0 < \beta < \infty$ $0 < \alpha < \infty$	$F(h_s) = e^{-\left(\frac{h_s}{\beta}\right)^{-\alpha}}$	$f(h_s) = \alpha\beta^{\alpha} h_s^{-(\alpha+1)} e^{-\left(\frac{h_s}{\beta}\right)^{\alpha}}$	$\beta\Gamma\left(1 - \frac{1}{\alpha}\right)$	$\beta^2\left[\Gamma\left(1 - \frac{2}{\alpha}\right) - \Gamma^2\left(1 - \frac{1}{\alpha}\right)\right]$

Notes: $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$

$$\gamma = \lim_{n \to \infty} 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n} - \ln(n) \cong 0.5772$$

LLED-CANCELLED-CANCELLED-CANC

In order to fit the distributions by the method of least squares, a non-exceedance probability must be assigned to each significant wave height. The data are first ranked by ascending significant wave height. If "m" is used to denote its rank, with m=1 corresponding to the smallest H_s and m=k corresponding to the largest H_s in a sample containing k storms, a simple measure of non-exceedance for each H_s value with a rank of m is m/(k+1). The method of least squares minimizes the sum of the squares of the vertical distances between the proposed theoretical cumulative distribution and m/(k+1).

SELECTION OF A DISTRIBUTION: A correlation coefficient, r, between 0 and 1 is printed with each residual table and is calculated by the following formula:

$$r = \sqrt{1 - \frac{\sum_{m=1}^{k} (Y_m - \hat{Y}_m)^2}{\sum_{m=1}^{k} (Y_m - \bar{Y})^2}}$$

where : k = the number of storms

Y_m = m/(k+1), the plotting formula for cumulative probability of non-exceedance

$\hat{Y}_m$ = $F(H_{sm})$, the probabilities estimated by the least squares curve

$\bar{Y}$ = 1/2, the mean cumulative probability

The distribution with the largest positive correlation coefficient is usually the best choice. If two or more of the distributions appear to fit equally well, the one that fits the larger values of H_s better is probably the best choice - especially for extrapolation purposes. This choice can be made by choosing the distribution with the smallest residuals, $\left|\hat{Y}_m - Y_m\right|$, for the larger values of H_s. The residuals can be examined in the residual table which is optionally printed.

RETURN PERIODS: The return period in years corresponding to a significant wave height of h_s is denoted by $RT(h_s)$ and is defined by the following formula:

$$RT(h_s) = 1/\lambda\ (1-F(h_s))$$

where λ is the Poisson lambda parameter. The return period is defined so that a $RT(h_s)$ year wave is expected to be exceeded once every $RT(h_s)$ years. A return period table is included in the output for each distribution.

The number of storms occurring per unit time is assumed to be a random variable best modeled by the Poisson distribution. The Poisson distribution is characterized by a mean value of λ, which in this case is the average number of storms per year. The value of λ is calculated as the number of storms divided by the period of record in years and is printed by the program. The probability density of the Poisson distribution is given by the following formula:

$$p(k) = \lambda^k e^{-\lambda}/k! \qquad k = 0, 1, 2, \ldots\ldots$$

PROGRAM APPLICATIONS: The probability distributions calculated by WAVDIST are useful in a wide variety of coastal engineering applications including formulation of design criteria for coastal structures, risk analysis, and estimation of long-term average or "expected" effects of varying wave heights. The output of WAVDIST also has been designed to support MACE programs BWLOSS1, BWLOSS2, and BWDAMAGE which deal with cost-effective optimization of rubblemound breakwaters (Smith, 1985).

PROGRAM LIMITATIONS: The user should be cautioned on "mixing populations." An example of this would be to input both tropical storm data and winter storm data into one running of the program. This violates the assumption that the data form an independent identically distributed random sample. The program does not account for possible depth limitations of the waves; however, the Weibull distribution can be adapted to account for these depth limitations (Isaacson and MacKenzie, 1981).

Since extrapolated values for the Weibull distribution are highly sensitive to the parameter α, it is recommended that care be taken when the estimated value of α falls outside a range between .75 and 3.0. This range of α values defines a family of curves including the exponential and Rayleigh distributions, which are widely used for modeling wave data. Any α values outside this range may lead to unsupported extrapolations.

PROGRAM AVAILABILITY: WAVDIST is available for the IBM PC on a 5-1/4-in. diskette in Microsoft BASIC and FORTRAN or as a printed program listing. WAVDIST may be obtained from Ms. Gloria J. Naylor at (601) 634-2581 (FTS 542-2581), Engineering Computer Programs Library Section, Technical Information Center, U.S. Army Engineer Waterways Experiment Station, P.O. Box 631, Vicksburg, MS 39180-0631. WAVDIST is also available as a listing in FORTRAN IV as implemented on the WES Honeywell DPS-8 mainframe. Alternate versions of WAVDIST and their functions are listed in the table below.

Program Title	Least Squares	Moments	Graphics
WAVDIST1	Yes	No	No
WAVDIST2	Yes	Yes	No
WAVDIST3	Yes	No	Yes
WAVDIST4	Yes	Yes	Yes

Questions regarding the technical details of WAVDIST and its applications may be addressed to Mr. Doyle Jones (601) 634-2069 (FTS 542-2069).

PROGRAM INPUT AND OUTPUT: The necessary program input and output are demonstrated by the following sample interactive session, which shows only the Extremal Type I results for brevity. The complete output provides similar results for the Weibull and Log-Extremal distributions.

SAMPLE PROBLEM: An analysis of 20 years of hindcast data revealed 22 events where peak zero moment wave heights exceeded 12.0 ft.

```
***************************************************************
*   WAVDIST1                    VERSION  5-86                 *
*                                                             *
*  WAVDIST1 IS AN INTERACTIVE PROGRAM WHICH FITS THREE        *
* TYPES OF LONG-TERM WAVE DISTRIBUTIONS TO HISTORICAL         *
* EXTREMAL WAVE HEIGHT DATA :                                 *
* (1) EXTREMAL TYPE I (ALSO GUMBEL OR FISHER-TIPPETT)         *
* (2) WEIBULL                                                 *
* (3) LOG EXTREMAL                                            *
*   THE PROGRAM ESTIMATES PARAMETERS FOR THE DISTRIBUTIONS    *
* BY THE METHOD OF LEAST SQUARES.  INPUT DATA CONSISTS OF     *
* THE SIGNIFICANT WAVE HEIGHT ASSOCIATED WITH EACH STORM,     *
* THE NUMBER OF STORMS, AND THE TIME INTERVAL IN WHICH        *
* THESE STORMS OCCURRED.  OUTPUT CONSISTS OF THE PARAMETERS,  *
* MEAN, AND VARIANCE FOR EACH DISTRIBUTION FUNCTION, A        *
* RETURN PERIOD TABLE, AND AN OPTIONAL TABLE OF RESIDUALS.    *
***************************************************************
```

```
IS DATA LOCATED IN A FILE (Y OR N) ? N

DO YOU WISH WAVE HEIGHT DATA STORED IN A DATA FILE (Y OR N) ? N

INPUT THE NUMBER OF STORMS YOU HAVE ? 22

INPUT THE TIME INTERVAL IN YEARS IN WHICH THE STORMS OCCURRED? 2Ø

INPUT THE SIGNIFICANT WAVE HEIGHT FOR EACH STORM - ONE AT A TIME
EXAMPLE: 7.6 <CR>
WAVE HEIGHTS UNITS SHOULD BE CONSISTENT

ENTER WAVE HEIGHT  1  ? 35
ENTER WAVE HEIGHT  2  ? 34.7
ENTER WAVE HEIGHT  3  ? 3Ø.9
ENTER WAVE HEIGHT  4  ? 29
ENTER WAVE HEIGHT  5  ? 28.1
ENTER WAVE HEIGHT  6  ? 24.9
ENTER WAVE HEIGHT  7  ? 24
ENTER WAVE HEIGHT  8  ? 23.9
ENTER WAVE HEIGHT  9  ? 23.3
ENTER WAVE HEIGHT  1Ø  ? 22.6
ENTER WAVE HEIGHT  11  ? 22.4
ENTER WAVE HEIGHT  12  ? 19.8
ENTER WAVE HEIGHT  13  ? 19.8
ENTER WAVE HEIGHT  14  ? 19.4
ENTER WAVE HEIGHT  15  ? 19
ENTER WAVE HEIGHT  16  ? 17.6
ENTER WAVE HEIGHT  17  ? 16.5
ENTER WAVE HEIGHT  18  ? 15.9
ENTER WAVE HEIGHT  19  ? 13.3
ENTER WAVE HEIGHT  2Ø  ? 12.3
ENTER WAVE HEIGHT  21  ? 12
ENTER WAVE HEIGHT  22  ? 12

PRINT RESIDUAL TABLES (Y/N) ? Y


DURING A PERIOD OF  2Ø  YEARS  22  STORMS OCCURRED
POISSON LAMBDA PARAMETER IS 1.1 STORMS PER YEAR

MEAN OF SAMPLE DATA           = 21.655
STANDARD DEVIATION OF SAMPLE =  6.878


                       LEAST SQUARES RESULTS


                       EXTREMAL TYPE I

F(hs)=Pr(Hs<hs)=  EXP(-EXP(-(hs-EPSI)/PHI))
EPSI         = 18.325
PHI          =  6.321
MEAN         = 21.973
VARIANCE     = 65.729
STD. DEV.    =  8.1Ø7
```

LLED-CANCELLED-CANCELLED-CANC

ORDER	XVALUE	YVALUE	YEST	DIFF
1	12.0	.0435	.0659	.02241
2	12.0	.0870	.0659	.02106
3	12.3	.1304	.0747	.05569
4	13.3	.1739	.1092	.06467
5	15.9	.2174	.2305	.01311
6	16.5	.2609	.2633	.00239
7	17.6	.3043	.3258	.02145
8	19.0	.3478	.4071	.05928
9	19.4	.3913	.4302	.03887
10	19.8	.4348	.4530	.01823
11	19.8	.4783	.4530	.02525
12	22.4	.5217	.5917	.06993
13	22.6	.5652	.6014	.03620
14	23.3	.6087	.6343	.02564
15	23.9	.6522	.6610	.00886
16	24.0	.6957	.6653	.03031
17	24.9	.7391	.7023	.03682
18	28.1	.7826	.8081	.02554
19	29.0	.8261	.8313	.00523
20	30.9	.8696	.8722	.00260
21	34.7	.9130	.9278	.01472
22	35.0	.9565	.9310	.02553

NON-LINEAR CORRELATION IS 0.9923153
SUM SQUARE RESIDUALS IS 0.0256283
STANDARD ERROR IS 0.0357968
MEAN SQUARE DEVIATION IS 0.0266467

RETURN PERIOD TABLE

YEAR	HS
5.0	28.48
10.0	33.18
25.0	39.16
50.0	43.60
100.0	48.01

REFERENCES:

Borgman, L. E. and Resio, D. T. 1977. "Extremal Prediction in Wave Climatology," Ports 77, Vol I., New York, N.Y., p. 394.

Isaacson, M. and MacKenzie, N. 1981. "Long-term Distributions of Ocean Waves: A Review," Journal of Waterway, Port, Coastal and Ocean Division, ASCE, Vol. 107, No. WW2. pp 93-109.

Smith, O. P. 1985. "Cost Effective Optimization of Rubble-Mound Breakwater Cross Sections," CERC Technical Report (in preparation), US Army Engineer Waterways Experiment Station, Vicksburg, MS.

U. S. Army Waterways Experiment Station, Coastal Engineering Research Center, "Reliability of Long-Term Wave Conditions Predicted With Data Sets of Short Duration", CETN-I-5, 1985, Vicksburg, MS.

LED-CANCELLED-CANCELLED-CANC