Coastal Engineering Technical Note

Beach-Fill Volume Required to Produce Specified Dry Beach Width

PURPOSE: To present a method developed by Dean (1991) that can be used to determine the volume of sand of arbitrary size required in a beach fill to produce a desired width of the subaerial beach.

BACKGROUND: The equilibrium profile of beaches is an important concept in beach-fill design. Bruun (1954) and Dean (1977; 1991) proposed that beach profiles develop a characteristic parabolic equilibrium beach shape given by

$$h = A\,x^{m} \tag{1}$$

where

h = still-water depth

x = horizontal distance from the shoreline

A = a dimensional parameter related to sediment characteristics

m = 2/3

Bruun (1954) showed that Equation 1 fit an extensive set of profiles from the Danish North Sea coast and a limited set of profiles from Mission Bay, CA. Dean (1977) also found that Equation 1 fit over 500 beach profiles collected by Hayden et al. (1975) along the U.S. east coast and the Gulf of Mexico. Almost half of these profiles were from the east coast of Florida. Equation 1 has been compared favorably with measured data from Poland and the Black Sea (Pruszak 1993) to New Zealand (Dean, Healy, and Dommerholt 1993).

Numerous studies have developed conceptual models to explain the form of Equation 1. An early paper by Keulegan and Krumbein (1949) concluded that a shoaling solitary wave would produce an equilibrium profile of the shape given by Equation 1 with $m = 2/5$. Dean (1977) considered dissipation of shallow-water linear waves and showed that if equilibrium is associated with wave energy per unit area of bed, then $m = 2/5$. However, if equilibrium is associated with wave energy per unit volume of the water column, then $m = 2/3$. Dean (1977) concluded that $m = 2/3$ led to the best fit with field data of profiles.

Dean (1987) has shown that A can be related to fall velocity of sediment by the following:

$$A = 0.067\,w^{0.44} \tag{2}$$

Fall velocity can be related to sediment diameter using equations developed by Hallermeier (1981a). Hallermeier (1981a) gives fall-velocity equations for a wide range of beach sand, temperature, and both fresh and salt water. For the case of common beach sand with diameters in the range of

0.15 mm to 0.85 mm and temperatures from 15 to 25° C, Hallermeier's equations can be reduced to give the following fall-velocity relationship:

$$w = 14\ D^{1.1} \tag{3}$$

w has units of centimeters/second and D is the diameter of the median-grain size of sediment with units of millimeters.

A can be expressed as

$$A = 0.21\ D^{0.48} \tag{4}$$

For example, 0.2-mm sand has a fall velocity of 2.4 cm/sec, which gives A = 0.97 m to the one-third power.

Early work on beach-fill design concentrated on the part of the beach that is visible; that is, the subaerial beach. However, it has been known for some time that the subaerial beach is only part of the beach system. When nourishing a beach, the entire nearshore profile down to an approximate closure depth must be nourished. Hallermeier (1981b) used laboratory data and limited field data from the Pacific Ocean and the Gulf of Mexico to develop a formula for computing closure depth. Birkemeier (1985) used an extensive data set measured at the Coastal Engineering Research Center's (CERC) Field Research Facility (FRF) located along the Atlantic Ocean in northeastern North Carolina to develop the following modification of Hallermeier's formula:

$$H = 1.75\ H_{s_{0.137}} - 57.9 \left(\frac{H^2_{s_{0.137}}}{gT_s^2} \right) \tag{5}$$

where

H = closure depth in meters

$H_{s_{0.137}}$ = extreme nearshore significant wave height exceeded 12 hrs per year in meters (i.e., 0.137 percent of the years)

T_s = period associated with $H_{s_{0.137}}$

For a Pierson-Moskovitz spectrum (Rijkswaterstaat 1986)

$$\left(\frac{H_s}{gT_s^2}\right)^{1/2} = 6.4 \ x \ 10^{-2} \tag{6}$$

yielding

$$d = 1.5 \ H_{s_{0.137}} \tag{7}$$

From the *Shore Protection Manual* (1984)

$$F_{0.137} = e^{\frac{-\left(H_{s_{0.137}} - H_{s_{min}}\right)}{\sigma}} \tag{8}$$

where

$F_{0.137}$ = 0.137 percent exceedence frequency

$H_{s_{min}}$ = $H_s - \sigma$

σ = significant wave height standard deviation

H_s = mean annual significant wave height

Therefore,

$$H_{s_{0.137}} = H_s + 5.6 \ \sigma \tag{9}$$

But $\sigma = 0.62 \ H_s$ from the *Shore Protection Manual* (1984)
so

$$H = 1.5 \ H_{s_{0.137}} = 6.75 \ H_s \tag{10}$$

Therefore, closure depth is equal to 6.75 times the mean average significant wave height. The relationship between significant wave height standard deviation and mean significant wave height is approximate when data are available, or should be calculated directly rather than estimated using the relationship from the *Shore Protection Manual* (1984).

Mean average significant wave height for any location in the United States is available from the series of "Wave Information Study" reports produced by CERC.

Equation 10 provides a simple method to calculate closure depth, but the best method to determine closure depth is to accurately measure the bathymetry of beach profiles over at least a year and determine the depth at which there are no significant variations of depth over the time of measurement.

If the size of beach-fill sediment is the same as that of the native beach, the beach profile after nourishment should be the same as that before

nourishment except extended seaward (Figure 1) in a "reverse" Bruun rule. The Bruun rule (Bruun 1962) basically says that as sea level rises, the shoreline retreats uniformly so as to maintain a constant equilibrium profile.

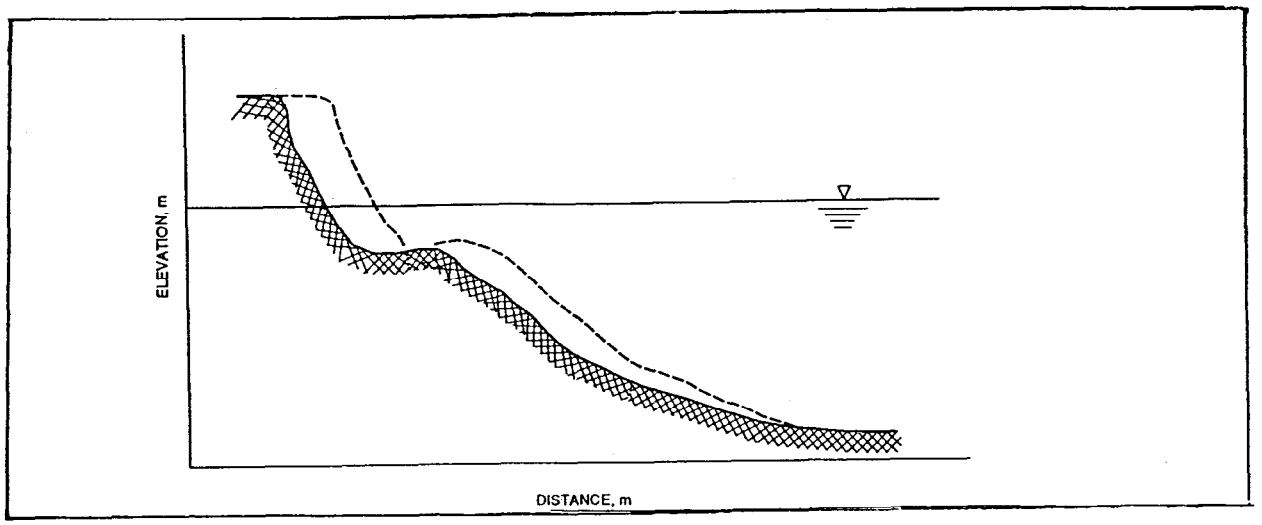

Figure 1. Displacement seaward of active profile following nourishment

Restoring the beach is, therefore, an inverse process where the entire profile has to be built seaward. Of course, this means that the volume of sediment needed to advance a beach is much more than the volume of fill on the subaerial beach. The *Shore Protection Manual* (1984) shows that when the height of beach berm is B (Figure 2) and the depth of closure is H, to build out a beach a distance Y requires a volume V of sediment per unit length of coastline given by

$$V = (B + H)\,Y \tag{11}$$

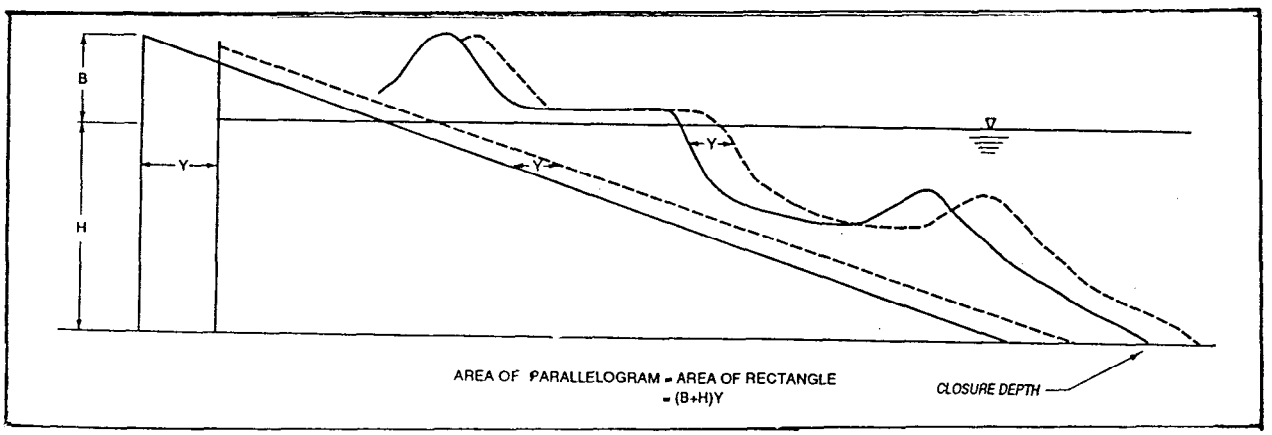

Figure 2. Volume of sediment per unit length of coastline resulting from fill placement

For example, if the fill-sediment size is the same as that of the native beach, berm height is 1.5 m, and closure depth is 6 m, to build out the dry beach by 30 m would require a volume of 225 m^3/m of beach. Equation 11 is easily derived by noting from Figure 2 that by moving the beach out a distance Y , a parallelogram is formed. The parallelogram can be shown to have an area identical to a rectangle (Figure 2) with a base equal to $B + H$. Therefore, the volume of the fill is equal to $(B + H)\ Y$.

GUIDANCE: A key design parameter is the width of the dry beach after the beach profiles have reached equilibrium. For the case of the fill sediment having a different size than the native sediment, Dean (1991) presents a method to determine the volume of sediment that must be placed per unit length of coast to achieve a desired dry-beach width after equilibrium. For the case of fill sediment identical to the native sediment and the distance out to closure depth being much larger than the width of the fill, Dean's method reduces to Equation 11.

Dean (1991) defines three basic types of nourished profiles. Figure 3 shows an intersecting profile, where the profile after nourishment intersects the native profile; a non-intersecting profile, where the nourished profile does not intersect the native profile before closure depth; and a submerged profile, where after equilibrium there is no dry beach. Dean (1991) shows that whether a profile is intersecting or non-intersecting is determined by the following inequalities:

$$Y\left(\frac{A_N}{H}\right)^{3/2} + \left(\frac{A_N}{A_F}\right)^{3/2} \langle\ 1,\ \textit{Intersecting profiles} \tag{12}$$

$$Y\left(\frac{A_N}{H}\right)^{3/2} + \left(\frac{A_N}{A_F}\right)^{3/2} \rangle\ 1,\ \textit{Non-intersecting profiles} \tag{13}$$

where

A_N = A value of the native sediment

A_F = A value of the fill sediment

For example, if Y equals 30 m, H equals 6 m, A_N equals 0.1, and A_F equals 0.09; then Inequality 13 is satisfied and the profiles are non-intersecting. However, if A_N equals 0.09 and A_F equals 0.1, then Inequality 12 is satisfied and the profiles are intersecting.

For the case of non-intersecting profiles where the fill sediment is finer than the native sediment (i.e., $A_F < A_N$), the volume of sediment that must be placed before there is any dry beach after equilibrium is

$$V = \frac{3}{5}\ H^{5/2}\left(\frac{1}{A_N}\right)^{3/2}\left(\frac{A_N}{A_F}\right)^{3/2}\left(\frac{A_N}{A_F} - 1\right) \tag{14}$$

where

V = volume of sediment in cubic m per m of beach

H = closure depth

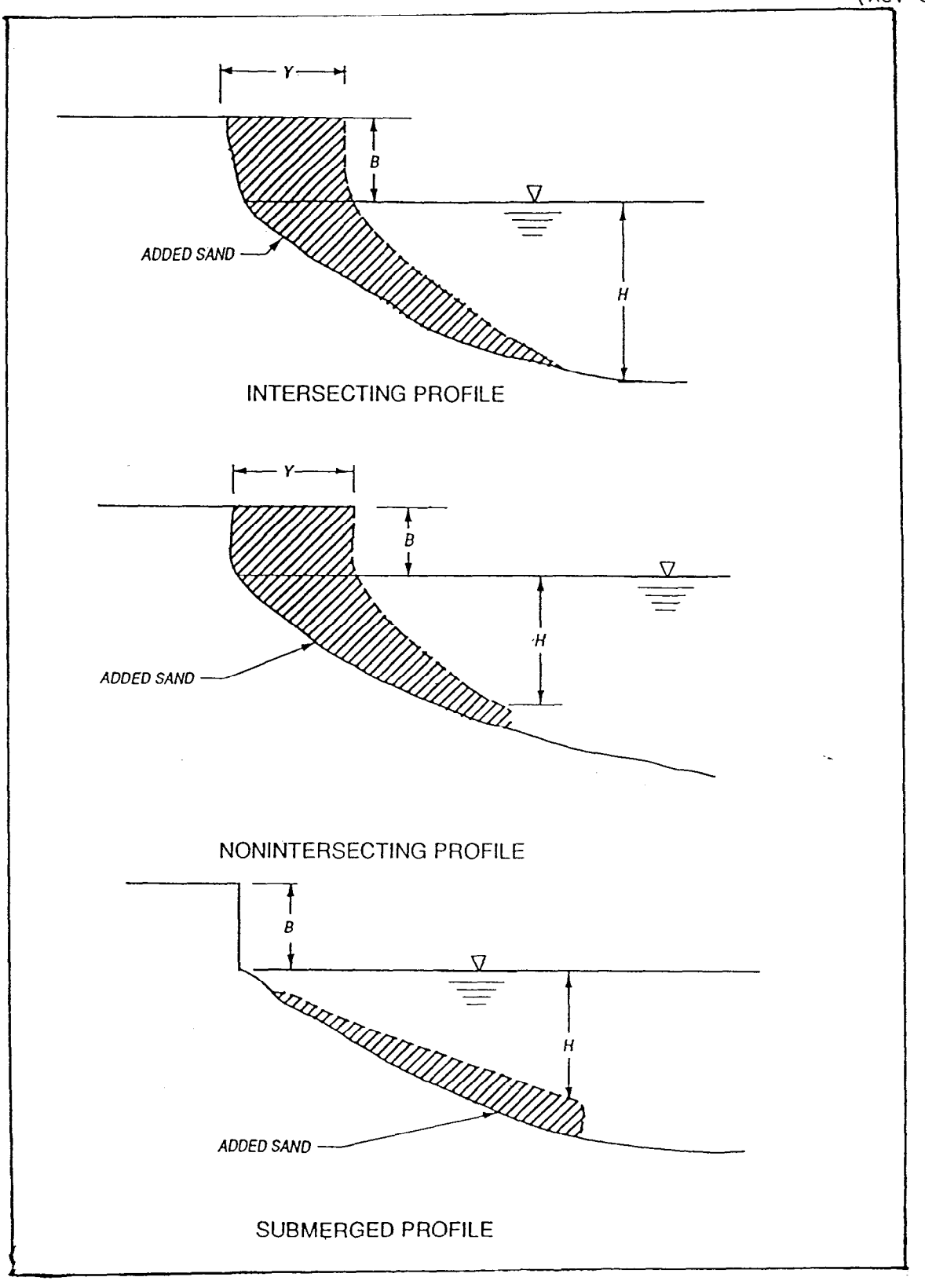

Figure 3. Three basic types of nourishment profiles (adapted from Dean (1991)

If the volume placed is less than given by Equation 14, the resulting profile is completely submerged.

For example, if closure depth H = 6 m, A_N = 0.1, and A_F = 0.09 , then V equals about 220 m^3/m of beach. This means that this volume must be placed before there is any dry beach after equilibrium.

For non-intersecting profiles (satisfies Inequality 13) with a dry beach after equilibrium (i.e., volume placed is equal to or exceeds that in Equation 14),

$$V = YB + \frac{3}{5} H^{5/2}\left[\left[\frac{Y}{H^{3/2}} + \left(\frac{1}{A_F}\right)^{3/2}\right]^{5/3} A_N - \left(\frac{1}{A_F}\right)^{3/2}\right] \tag{15}$$

If closure depth H = 6 m, berm height B = 1.5, and the sediment diameters for the native beach and fill beaches are 0.25 mm and 0.2 mm respectively, what is the volume required to extend the width of the beach 30 m after equilibrium? From Equation 4, A_N = 0.108 and A_F = 0.097 . Equation 15 yields a volume V , of about 450 m^3/m of beach length.

For intersecting profiles (generally the case when fill sand is coarser than native, but Inequality 12 defines), the volume V , required to advance the dry beach a distance Y after equilibrium is given by

$$V = BY + \frac{\frac{3}{5} A_N Y^{5/3}}{\left[1 - \left(\frac{A_N}{A_F}\right)^{3/2}\right]^{2/3}} \tag{16}$$

If the berm height is 1.5 m, the native and fill beaches are 0.2 mm and 0.25 mm, respectively (so the fill sand is coarser than the native), to extend the dry beach 30 m after equilibrium requires that A_N = 0.097 and A_F = 0.108 . Equation 16 yields V equals about 105 m^3/m of beach. Note that the closure depth is not contained in Equation 16 since the profiles intersect before closure depth. The problem is essentially the same as that in the previous paragraph except that the grain sizes of the native and fill sands are reversed. Fill with coarser sediment than the native requires a substantially smaller volume than the fill with finer sediment than the native, as expected. Recall that when native and fill sediments are the same, the fill volume of 225 m^3/m is between the volumes determined using Equations 15 and 16.

A shortcoming of the approach presented is that it represents the sediment on the profile by a single median-grain size. Actually, it is well known that sediment size generally decreases as one moves seaward. Dean and Abramian (1993) modify the approach given above to allow a decrease in sediment size along the profile.

<u>PROCEDURE</u>: Berm height B and width of the dry beach after equilibrium Y are given. Closure depth H equals 6.75 times the mean average significant wave height. Sediment sizes for the native and fill beaches are known.

A_N and A_F are determined using Equation 4. Inequalities 12 and 13 are used to determine if the profiles are intersecting or non-intersecting. If

they are intersecting, Equation 16 is used to calculate the volume of sediment required to advance the shoreline a distance Y. If the profiles are non-intersecting, Equation 15 is used to calculate Y. Equation 14 can be used for non-intersecting profiles to determine the volume that must be placed to make Y greater than zero. Examples are provided for each case in the "Guidance" section.

ADDITIONAL INFORMATION: For further information contact Dr. James R. Houston, Director, Coastal Engineering Research Center, U. S. Army Engineer Waterways Experiment Station, at (601) 634-2000.

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