Coastal Engineering Technical Note

BEACH-FILL TRANSITIONS

PURPOSE: To present a method for determining beachfill transition lengths, fill quantities, and costs for optimum design.

GENERAL: Generally, the new shore alignment of a beach restoration project parallels the existing shore alignment. Transition zones between the terminal points of the beachfill and the unrestored updrift and downdrift beaches are usually required, unless groins, jetties, or other shoreline projections are used to compartment the fill. When the beachfill is placed on a normally straight beach, the orientation of the transition shoreline will differ from the natural shore alignment (see Figure 1) causing different erosion rates for the transition section than those experienced by the natural shore alignment. The U.S. Army Engineer District, Wilmington (1973), recognizing that the rate of littoral transport is dependent on the relative angle between the breaker angle and the shoreline transition angle, developed an approximate method for designing beachfill transitions by evaluating a number of transition plans and determining which plan produces the optimum improvement. Different transition angles and corresponding transition lengths were used to compute the total annual costs for initial beach restoration and periodic nourishment. Benefits are assumed to stay constant for all plans so the least costly transition orientation would be the optimum plan.

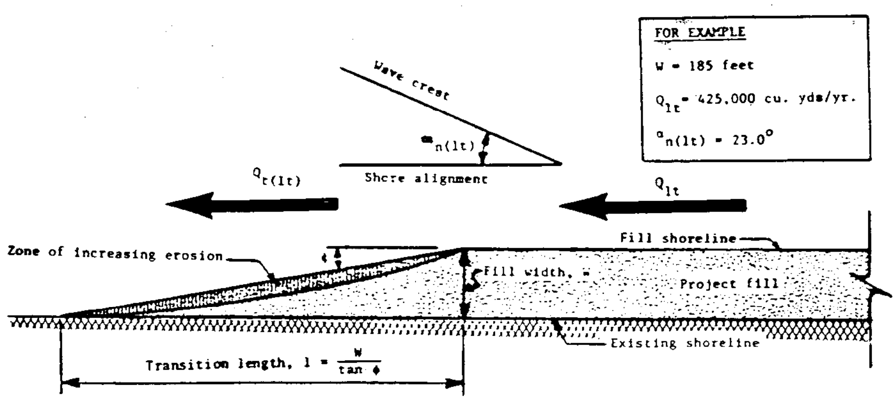

Figure 1. Downdrift Beachfill Transition
(Predominant direction of littoral transport)

PROCEDURE: The first step in optimization is to determine the expected loss of beachfill material from the different transition sections. This loss includes the normal erosion and the loss of fill material out of the restored area by littoral transport. The erosion rate and littoral transport rate for the non-transition shoreline needs to be previously determined. The normal beach erosion varies with the length of the transition of the project fill and the angle of transition. The *Shore Protection Manual* (1977) (Vol. I, Section 4-53) gives equations and procedures for computing longshore littoral transport. The only factor in these equations that differ between the beachfill and the adjacent transition zone is the angle at which the waves break on the respective shore alignments. The longshore transport rate at the transition, Q_t, can be related to the longshore transport rate of the beachfill, Q, by the expression:

$$Q_t = RQ \qquad (1)$$

where

$$R = \frac{\sin(2\alpha_t)}{\sin(2\alpha_n)} \qquad (2)$$

and α_t = breaker crest angle relative to the transition,

and α_n = breaker crest angle relative to the existing shoreline or beachfill.

For the downdrift transition (transport right to left in Figure 1)

$$\alpha_t = \alpha_n + \phi \qquad (3)$$

and

$$R_{1t} = \frac{\sin 2(\alpha_{n(1t)} + \phi)}{\sin 2\,\alpha_{n(1t)}} \qquad (4)$$

where ϕ is the transition angle relative to the existing shoreline or beachfill. Thus the longshore transport rate at the downdrift transition is given by Equations (1) and (2) as:

$$Q_{t(1t)} = Q_{1t}\left(\frac{\sin 2(\alpha_{n(1t)} + \phi)}{\sin 2\alpha_{n(1t)}}\right) \qquad (5)$$

In this case, because the angle α_t is increased by the value of ϕ, R is greater than one; thus $Q_{t(1t)}$ is greater than Q_{1t}.

For the case of transport reversal or updrift transition (transport shown left to right in Figure 2):

$$\alpha_t = \alpha_{n(rt)} - \phi, \qquad (6)$$

$$R_{rt} = \frac{\sin 2(\alpha_{n(rt)} - \phi)}{\sin 2\,\alpha_{n(rt)}} , \qquad (7)$$

and

$$Q_{t(rt)} = Q_{rt}\left(\frac{\sin 2(\alpha_{n(rt)} - \phi)}{\sin 2\,\alpha_{n(rt)}}\right). \qquad (8)$$

In this case, because the angle α_t is decreased by the value of ϕ, R is less than one, thus $Q_{t(rt)}$ is less than Q_{rt}.

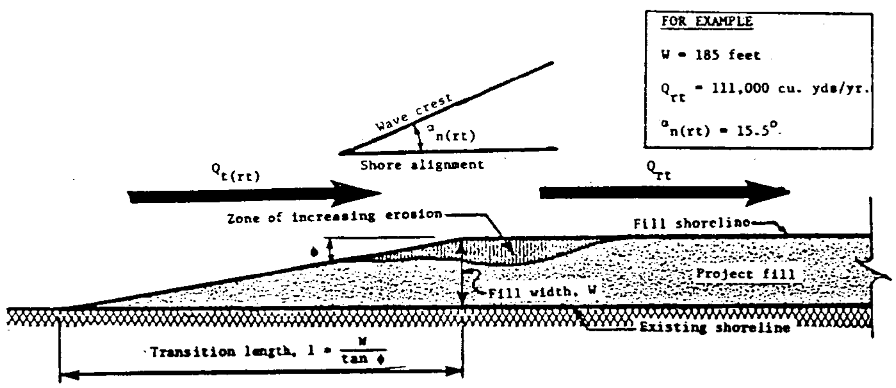

Figure 2. Downdrift Beachfill Transition
(Reversal direction of littoral transport)

With different transition angles (ϕ), factors which influence the amount of material required and the cost of the transition fill vary in the following manner:

a. As ϕ decreases, the length of the fill transition increases resulting in an increase in the initial amount of beachfill required.

b. As ϕ decreases, the overall quantity of periodic beach nourishment increases, due to the necessity of maintaining a longer length of project shoreline.

c. As ϕ decreases, the expected rate of erosion per unit length of the transition decreases, due to the more normal alignment of the shoreline transition.

As a result of these varying effects, a sufficient number of transition angles needs to be used until the minimum transition cost is achieved. It should be determined if a terminal groin would provide a more economical solution than the fill transition.

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Example computation is for a transition angle, relative to the existing shoreline, ϕ = 1.0 degrees. For computed values using other value of ϕ, see Table 1.

GIVEN:

1. The authorized fill project advances the shoreline seaward of its present location a width of 185 feet and the average amount of fill is 0.91 cubic yards per square foot.

2. Annual erosion rate for existing shoreline is 8.78 cubic yards per lineal foot.

3. Normal littoral transport from right to left, Q_{lt}, is 425,000 cubic yards per year and average breaker angle of waves that generate right to left littoral transport, $\alpha_{n(rt)}$, is 23 degrees (see Figure 1).

4. Normal littoral transport from left to right, Q_{rt}, is 111,000 cubic yards per year and average breaker angle of waves that generate left to right littoral transport is 15.5 degrees (see Figure 2).

5. Assume a cost of $2.50 per cubic yard for beachfill and periodic nourishment amortized for 50 years @ 7 5/8 per cent per year. This results in an annual cost of $0.196 /cu. yd.

FIND (For $\phi = 1.0^{o}$):

1. Transition length (feet)
2. Initial transition fill (cu. yds.)
3. Normal annual erosion (cu. yds. per year)
4. Increase in annual erosion from littoral transport from the left (cu. yds. per yr.)
5. Increase in annual erosion from littoral transport from the right (cu. yds. per yr.)
6. Total amount of annual erosion (cu. yds. per yr.)
7. Annual cost of beach nourishment ($ per yr.)
8. Annual cost of initial transition fill ($ per yr.)
9. Total annual cost of transition ($ per yr.)

SOLUTION:

1. Length of transition, $\ell = \frac{W}{\tan \phi} = \frac{185 \text{ ft}}{\tan 1.0°} = \frac{185}{0.0175} = 10{,}600$ ft.

2. Initial transition fill = 10,600 x 185 x 1/2 x 0.91 cu. yds./sq. ft. = 892,500 cu. yds.

3. Normal annual erosion = 8.78 cu. yds./lin. ft. x ℓ.
= 8.78 x 10,600 ft. = 93,100 cu. yds./yr.

4. Increase in annual erosion from littoral transport from the left = $Q_{lt} - Q_{t(lt)} = Q_{lt} - (R_{lt} \times Q_{lt})$.

$$R_{lt} = \frac{\sin 2(\alpha_{n(lt)} + \phi)}{\sin 2\,\alpha_{n(lt)}} = \frac{\sin 2(23° + 1.0°)}{\sin 2(23°)} = \frac{\sin 48°}{\sin 46°} = \frac{0.7431}{0.7193} = 1.033.$$

$Q_{t(lt)} = R_{lt}$ x 425,000 cu. yds./yr. = 1.033 x 425,000 = 439,000 cu. yds./yr.

Increase in erosion = 439,000 - 425,000 = 14,000 cu. yds./yr.

5. Increase in annual erosion from littoral transport from the right = $Q_{rt} - Q_{t(rt)} = Q_{rt} - (R_{rt} \times Q_{rt})$

$$R_{rt} = \frac{\sin 2(\alpha_{n(rt)} - \phi)}{\sin 2\,\alpha_{n(rt)}} = \frac{\sin 2(15.5° - 1.0°)}{\sin 2(15.5°)} = \frac{\sin 29°}{\sin 31°} = \frac{0.4848}{0.5150} = 0.941$$

$Q_{t(rt)} = R_{rt}$ x 111,000 cu. yds./yr. = 0.941 x 111,000 = 104,500 cu. yds./yr.

Increase in erosion = 111,000 - 104,500 = 6,500 cu. yds./yr.

6. Total amount of annual erosion = 93,100 + 14,000 + 6,500 = 113,600 cu. yds./yr.

7. Annual cost of beach nourishment = 113,600 cu. yds./yr. x $2.50/cu. yds. = $284,000/yr.

8. Annual cost of initial transition fill = 892,500 cu. yds./yr. x $0.196 per cu. yd. = $175,000 per year.

9. Total annual cost = $284,000 + $175,000 = $459,000.

Other computations were made to determine the average annual cost of constructing and maintaining transitions having transition angles of 1.5°, 2°, 2.5°, 3°, and 3.5°. A summary of these computations is given in Table 1.

Table 1. Optimization of Downdrift Transition Fill

Line No.	Transition Angle (Φ)	1.0°	1.5°	2.0°	2.5°	3.0°	3.5°
1	Length of Transition (ℓ = 185/tan Φ) (ft)	10,600	7,060	5,300	4,240	3,530	3,020
2	Initial Transition Fill (cu. yds.)	892,500	594,500	446,300	357,000	297,200	254,300
3	Normal Erosion (8.78 x line 1) (cu. yds./yr)	93,100	62,000	46,500	37,200	31,000	26,500
4	Transport from Left (R_{lt})	(1.033)	(1.049)	(1.065)	(1.080)	(1.095)	(1.110)
	$Q_{t(lt)}$ = (R_{lt} x 425,000)	(439,000)	(445,800)	(452,600)	(459,000)	(465,400)	(471,800)
	Incr. in Erosion ($Q_{t(lt)}$-425,000) (cu.yds/yr)	14,000	20,800	27,600	34,000	40,400	46,800
5	Transport from Right (R_{rt})	(0.941)	(0.912)	(0.881)	(0.851)	(0.821)	(0.790)
	$Q_{t(rt)}$ = (R_{rt} x 111,000)	(104,500)	(101,200)	(97,800)	(94,500)	(91,100)	(97,700)
	Increase in Erosion (111,000 -$Q_{t(rt)}$) cu.yds/yr)	6,500	9,800	13,200	16,500	19,900	23,300
6	Total Amount of Erosion (lines 3+4+5)	113,600	92,600	87,300	87,700	91,300	96,600
7	Annual Cost of Nourishment ($) 1/	284,000	232,000	218,000	219,000	228,000	242,000
8	Annual Cost of Initial Transition Fill ($) 2/	175,000	117,000	87,000	70,000	58,000	50,000
9	Total Annual Cost ($) (lines 7+8)	459,000	349,000	305,000	289,000	286,000	292,000

1/ Cost is based on a unit cost of $2.50 per cubic yard.

2/ Cost is based on a unit cost of $2.50 per cubic yard, interest rate of 7 5/8 percent, and an amortization period of 50 yrs. (capital recovery factor = 0.07824). This results in an annual cost of $0.196 per cu. yd.

CONCLUSION: According to the optimization analysis in this example, a transition angle of 3.0° provides the least costly orientation of a down-drift transition. However due to the relatively long (over 3,500-foot) transition, it may be more practical to compartment the beachfill material with a groin.

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ADDITIONAL INFORMATION: For addition information contact the Coastal Engineering Studies Section, Wilmington District (919) 343-4778 or FTS 671-4778.

REFERENCES:

U.S. ARMY, CORPS OF ENGINEERS, COASTAL ENGINEERING RESEARCH CENTER, *Shore Protection Manual*, 3rd ed, Vols. I, II, and III, Stock No. 008-022-00113-1, U.S. Government Printing Office, Washington, D.C., 1977, 1,262 pp.