US Army Corps
of Engineers

Grid Development for Modeling Two-Dimensional Inlet Circulation

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PURPOSE: The Coastal Engineering Technical Note (CETN) herein provides guidance for developing computational grids for numerical modeling of water level and current in and around inlets.

This CETN applies to grid development for two-dimensional (depth-integrated) triangular finite-element and rectilinear finite-difference models. Topics covered are grid resolution, representation of structures and boundary conditions, and considerations for stability and computational efficiency.

BACKGROUND: The U.S. Army Corps of Engineers has responsibility for constructing and maintaining inlet navigation channels and associated structures such as jetties. Understanding the flow field in the vicinity of inlets is required for designing navigation channel and jetty modifications. With the power of desktop computers rapidly increasing, hydrodynamic modeling of inlet systems lies within the capability of the field engineer. Software packages such as the Surface Water Modeling System (SMS), which is available to U.S. Army Corps of Engineers Districts, provide sophisticated and convenient user interfaces for grid development. The information in this CETN is presented to guide engineers in grid development for inlet applications. Well-designed grids form the basis for accurate calculation of water level and velocity, and many decisions must be made in the grid-generation process. Consideration must be given to domain extent, resolution of specific features (such as channels, shoals, and structures), specification of boundaries, bay coverage, and numerical stability.

DOMAIN EXTENT: Grid development should be conducted with knowledge of the hydrodynamic system being modeled and the processes of importance for a particular study. Factors that play a role in the grid development include tidal range, wind forcing, bay size and geometry, tributary inflow, presence of tidal flats, channels, shoals, structures, and perhaps atmospheric pressure if hurricanes are of consideration. The physical features of the site (shoals, channels, etc.) must be represented such that the model will respond correctly to the driving forces. A wide range of coastal inlet systems exist, and the modeler must understand the relative contributions of the driving forces active within a project area and the system response to those forces.

The domain extent over the ocean is selected based on several types of information. The first consideration is the area of interest for a particular application. If a study requires hydrodynamic calculations only in a local region, such as in the direct vicinity of an inlet, then the ocean domain may be relatively small. However, if the influence of one inlet on another is to be studied, then the domain extent must be large enough to include the two inlets. In any case, domain boundaries must be sufficiently far from the area of interest so that localized inaccuracies near boundaries do not influence calculations in the study area.

A second consideration for domain extent is the conditions under which the model will be run. For instance, if the application considers only tide and fair-weather (no large storms) simulations, a smaller grid may suffice in comparison to the grid needed for simulation of storms, such as hurricanes or large extratropical (northeaster) storm systems. If severe large-scale storms are to be modeled, a large domain is required to accurately calculate storm surge and ocean response (Blain et al. 1995). In representing hurricanes, storm surge may be underpredicted if the domain is not sufficiently large to simulate the large-scale storm-driven processes.

Two types of ocean boundaries are commonly applied for modeling coastal areas. One type applies time series of water-level forcing constructed from astronomical tidal constituents at the open ocean boundary. Only the tidal constituents specified at the boundary are propagated into the system from the ocean boundary. Wind and atmospheric pressure forcing can be added over the model domain in addition to the tide. This ocean boundary condition is preferable if accurate water level and current values seaward of the inlet are required. Ocean boundaries are specified far from the area of interest, and the tide is propagated over a large domain that can include the abyssal ocean and continental shelf, as well as the inlet and bay of interest. Large elements or cells can be specified over the ocean with size decreasing toward the inlet so that a large area can be covered without adding much computational expense. Tidal-constituent databases for the East Coast, Gulf of Mexico, and Caribbean Sea, and West Coast of the United States and Eastern North Pacific Ocean are described in Scheffner (1994) and Scheffner (1995), respectively. These databases contain tidal amplitudes and phases for application as oceanic tidal forcing.

The second type of ocean boundary applies a time-series water level over a much smaller ocean boundary. Gauge data can be applied as the water-level forcing, and these data contain motion propagated from the far field. This type of boundary condition can be applied if the area of interest lies within the bay and coastal currents are not of concern. Flow structure in the vicinity of the inlet entrance is more accurately calculated if the tidal-phase differences on the ocean boundary are taken in account. If horizontal patterns of the current on the seaward side of the inlet are of interest, a large-domain tidal constituent boundary condition (as described in the preceding paragraph) will give more accurate results.

Often, the question is posed as to how much of the back bay should be included in the grid. Although the modeling focus may be for a limited region, such as within an inlet channel or at a dredged-material disposal site either in the bay or in the ocean, accuracy of the model may be compromised if the back bay boundaries are located close to the inlet or in regions of strong current velocity. Some bays are sufficiently small such that inclusion of the entire bay does not add a significant computational expense. It is preferable to include an entire bay in the model domain so that the storage capacity of the bay is accurately represented. For large bays with one or more tributaries, discharge should be specified for any tributaries where data are available or where reasonable estimates can be made.

VERTICAL DATUM SELECTION: All bathymetry, depth, and water-surface elevations are referenced to the same vertical datum, so one that is relatively consistent over the model domain should be selected. Often, tide gauges are surveyed into a National Geodetic Vertical Datum (NGVD), which is a reasonable datum selection for modeling because it is generally spatially consistent within a localized region. However, any datum that is relatively consistent over the local area of interest can be applied. The ambient depths within a domain can be referenced to NGVD or other datum through local tidal datums. Tidal datums can vary significantly over a model domain because the tidal wave can undergo both damping and shoaling as it propagates over the continental shelf, through the inlet, and into a bay or estuary. For example, significant changes in tidal datums can occur over relatively short distances, such as through inlets. A tidal datum such as Mean Lower Low Water can be different on either side of an inlet because attenuation of the tidal signal through the inlet changes the tidal range within the bay, as compared with the coastal ocean. In some situations, the tidal range within an embayment can be increased, owing to entrance geometry (usually wide) and response of the embayment to tidal forcing. This increase in tidal range spatially modifies the tidal datums. In addition, Mean Sea Level can vary several centimeters between an embayment and the coastal ocean (Hicks 1984). All water-level datums vary spatially, but some vary more than others within a given region. Thus, a datum that is (nearly) spatially consistent over the computational domain should be selected for vertical reference.

BATHYMETRY: The quality of the bathymetric data used in grid generation plays a significant role in the accuracy of the hydrodynamic calculations. Because the bottom topography can change significantly over a relatively short time, bathymetric data collected closest to the simulation time should be incorporated in grid generation. In particular, high-quality data are needed in the vicinity of an inlet, including the inlet throat, ebb and flood shoals, and adjacent channels. Variations in the bottom topography and the overall geometry of features strongly influence the flow and variation in water elevation. For instance, a long and narrow inlet may significantly damp the tidal elevation as the wave propagates through the inlet. In the case of Aransas Pass, Texas, which is a Federally maintained deep-draft channel, the tidal amplitude is diminished by 40 percent as it propagates 2 km from the Gulf of Mexico into Corpus Christi Bay (Militello 1998). Accurate calculation of water-level variation within the bay requires a good representation of the inlet geometry (variations in width and length, shoreline position, bottom topography).

The velocity can vary across an inlet because of changes in ambient depth. Ponce De Leon Inlet, located on the Atlantic Coast of Florida, exhibits cross-sectional flow variations that can only be reproduced using accurate bathymetry. Incorrect depth specifications would result in underprediction or overprediction of the velocity, and the flow distribution across the inlet would not be calculated accurately. At Ponce De Leon Inlet, the south spit is presently migrating northward, causing shoal formation in the southern half of the inlet. The northern side of the inlet is experiencing scour caused by high-velocity flow. High-quality bathymetric data sets collected by the SHOALS (Lillycrop, Parson, and Irish 1996) system have allowed for accurate calculation of cross-sectional variation in the inlet flow field, as shown in Figure 1.

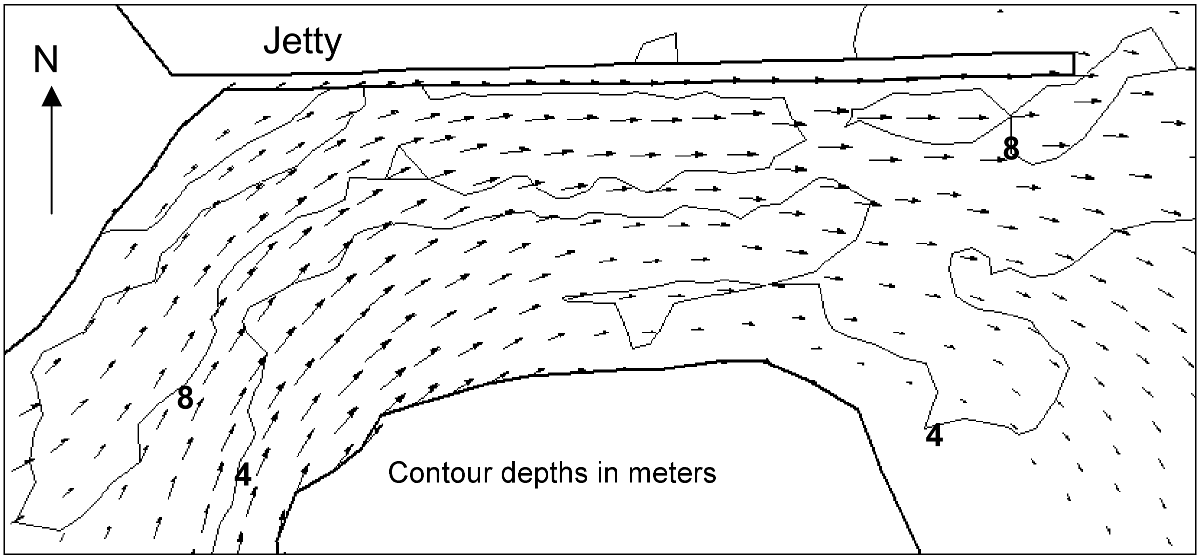

Figure 1. Calculated current vectors in Ponce De Leon Inlet, Florida, overlain on depth contours (4-m intervals)

Away from the area of interest, accuracy of bathymetry is less important, but inputting into the grid the most accurate and representative bathymetric data for the time period being modeled is recommended. In coastal and shelf areas, the depth governs shoaling and propagation speed of the tidal wave. Thus, tidal amplitude and phase are influenced by the coastal and offshore bathymetry.

For inlet studies, back-bay regions distant from an inlet may not require detailed bathymetry. These regions are often included in grids for accurate calculation of water storage, and details of their hydrodynamics may not be of concern. Back-bay geometry and water storage can influence the phase difference between the water level and current in and around an inlet. In regions distant from the inlet, the main channels and relatively larger scale features of the back bay should be included, but small features and minor channels can often be represented without much detail. In this manner, the influence of the back bay on the inlet hydrodynamics can be taken in account.

GRID RESOLUTION: Variable grid resolution is a convenient feature for allowing the modeler to optimize calculation detail with computational efficiency. In general, grids should be coarse in the open ocean, but with finer resolution over changes in depth, such as the continental shelf. Fine resolution should be specified in the direct areas of interest and where relatively large spatial gradients in depth are present (if they will influence the hydrodynamics in the study area). For flow around structures, such as jetty tips, fine resolution is required. Figure 2 gives an example of variable resolution in a finite-element mesh developed for Ponce De Leon Inlet. Element sizes are small within the inlet and in the back bay and increase in size with distance from the inlet. The fine resolution within the back bay (in regions relatively close to the inlet) facilitates accurate flow calculations through the complex system of channels and shoals. Figure 3 shows detail of the grid at the inlet. High resolution is specified within the inlet, which allows for detailed flow calculations along and across the inlet, as well as near the jetty.

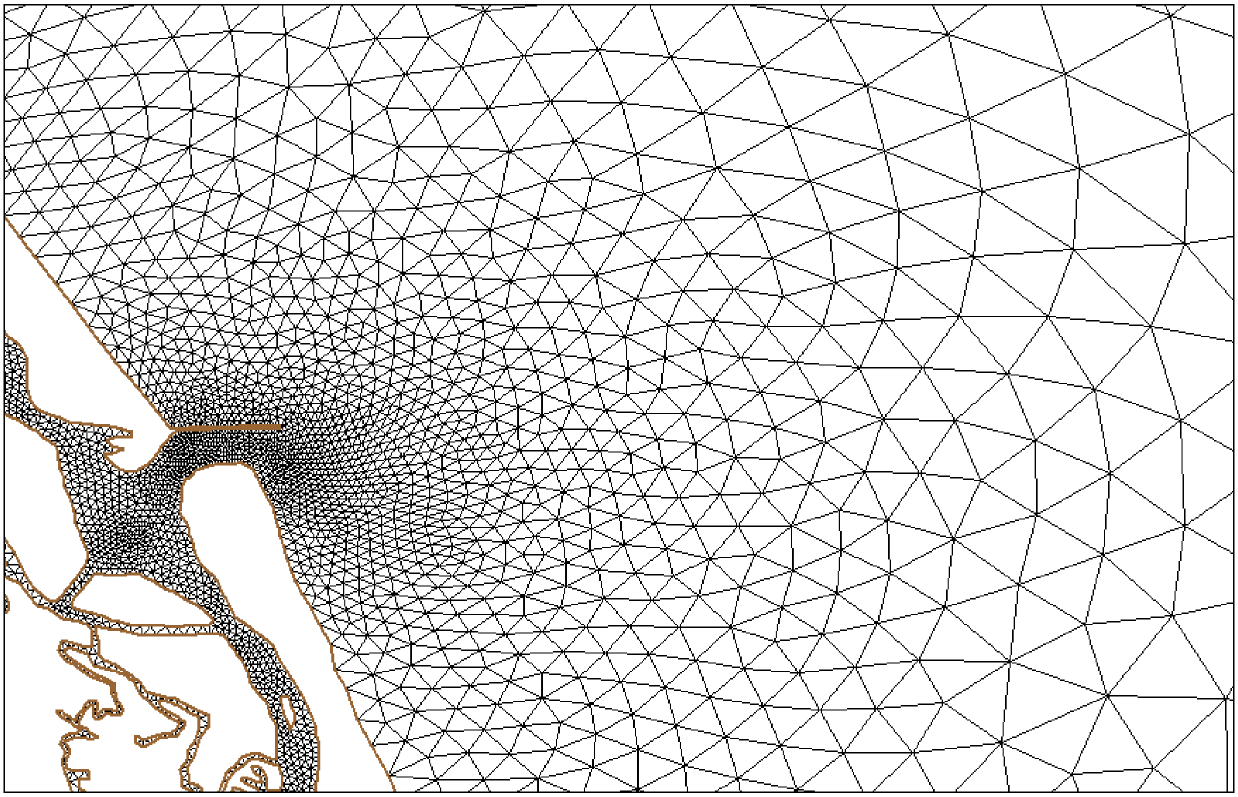

Figure 2. Graded finite-element mesh, Ponce De Leon Inlet, Florida

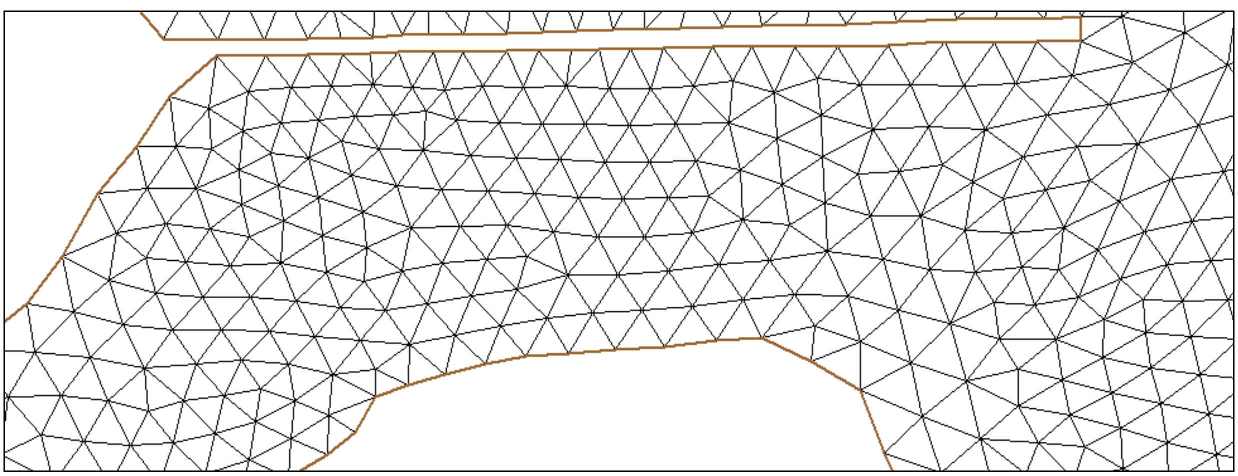

Figure 3. Detail of finite-element mesh in inlet, Ponce De Leon Inlet, Florida

Grid resolution must be sufficient to distinguish major channels from adjacent shallower areas. If channels are not explicitly resolved, the flow through them will not be calculated accurately. Navigation channels should usually be modeled with one to three elements (or cells) across the channel, depending on the level of interest in the calculated velocities within the channel. Higher resolution within the channel can be specified if details of the flow field are of interest.

Grid resolution can vary spatially, and grading between coarse and fine resolution should be done with regard to transition between element or cell areas. If large variations in grid resolution are specified over a small region, accuracy of the calculations can be compromised. A general rule of thumb is that adjacent elements should not differ in size by more than 50 percent (Donnell et al. 1996). Figure 2 shows an appropriate transition between element sizes within a finite-element mesh. Note that adjacent elements have similar size.

For finite-element grids, element shape can influence accuracy of the model computations. Ideally, all elements in a grid should be equilateral to minimize truncation error. However, most grids have elements that are not equilateral, particularly near shorelines and structures. To minimize computational errors, element angles (interior angles of the triangular element) must be no less than 30 deg.[1] The SMS package contains a grid quality-checking utility that will tag elements that do not meet specified shape criteria.

Cell shape in rectilinear finite-difference grids can influence computational accuracy. The cell shape can be described by the aspect ratio *R*, which is calculated by

$$R = \frac{\Delta s_1}{\Delta s_2} \tag{1}$$

where Δs_1 and Δs_2 are the cell side lengths parallel to the two horizontal axes and $\Delta s_1 \geq \Delta s_2$. In the area of interest for a given application, such as in the vicinity of an inlet, *R* should be limited to maximum values of 2 or 3 to minimize computational errors.

GRID RESOLUTION AND TIME-STEP: Greater grid resolution increases computation time by two means: (a) increased numbers of nodes and elements (or cells) require more calculations than fewer numbers of nodes and elements (or cells), and (b) higher resolution requires a smaller time-step to maintain numerical stability. For models that use explicit-solution schemes, a stability criterion dependent on the Courant number can be applied to estimate the computational time-step. The Courant number C_r is given by

$$C_r = \frac{\Delta t \sqrt{gh}}{\Delta s} \tag{2}$$

where

Δt = time-step
Δs = grid spacing
g = acceleration because of gravity
h = ambient depth

For stability, the theoretical upper limit of the Courant number is 1. However, in practical application, model stability requires that $C_r < 1$. Optimal Courant numbers depend on the specific numerical approach applied in a given model, and the value of the Courant number

[1] Personal Communication, 1998, J. J. Westerink, Codeveloper of ADvanced CIRCulation (ADCIRC) numerical model), Department of Civil Engineering, University of Notre Dame.

applied influences both stability and accuracy. Implicit models allow larger time-steps to be taken, and the Courant number is not applicable for these models unless the nonlinear terms are expressed in explicit form.

Example

Given a grid spacing of 80 m and an ambient depth of 10 m, one estimates the required time-step for an explicit model. The estimate is calculated by rearranging Equation 2 as

$$\Delta t = \frac{C_r \Delta s}{\sqrt{gh}} \tag{3}$$

Assuming a maximum Courant number of 1, the estimated time-step is

$$\Delta t = \frac{(1)(80)}{\sqrt{(9.8)(10)}} = 8\,\mathrm{sec}$$

Areas having fine resolution in relatively deep water often dictate the maximum time-step that can be applied to achieve stability. For instance, a grid will be detailed within an inlet so that the flow field can be well represented. However, a deep section in an inlet or channel, such as a scour hole, may limit the time-step to a few seconds. The engineer must weigh computational expense versus resolution in specific areas. Also, invoking the nonlinear terms (advection and finite-amplitude terms) in the calculations will decrease the allowable time-step. These terms are often necessary to accurately calculate flow fields, particularly near the entrances of inlets where large gradients in the flow can occur.

ADDITIONAL INFORMATION:

Questions about this CETN can be addressed to Dr. Adele Militello at (601) 634-3099 or e-mail at *a.militello@cerc.wes.army.mil*. For information about the Coastal Inlets Research Program, please contact the Program Manager, Mr. E. Clark McNair, at (601) 634-2070 or e-mail at *mcnairc@ex1.wes.army.mil*. For information about the Surface-Water Modeling System (SMS), please contact Ms. Barbara Donnell at (601) 634-2730 or e-mail at *donnelb@mail.wes.army.mil* or Mr. Bill Boyt at (601) 634-3249 or e-mail at *boytw@mail.wes.army.mil*.

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