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Mathematical Model for Rapid Estimation of Infilling and Sand Bypassing at Inlet Entrance Channels

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PURPOSE: The Coastal and Hydraulics Engineering Technical Note (CHETN) herein describes a mathematical for rapid estimation of rates of infilling and bypassing at inlet entrance channels located on sandy or gravel shores. Infilling is assumed to occur by cross-channel transport. The calculation procedure requires information typically available or estimated in coastal navigation projects and is intended to provide guidance for projects where detailed studies cannot be performed. The procedure can be applied to any channel that meets the basic assumptions.

BACKGROUND: Navigation channels issuing through an inlet entrance intercept sediment moving alongshore. The longshore transport may be generated by wave- and wind-generated currents, and by the longshore component of the flood-tidal current entering the channel. Sediment (assumed to be predominantly sand or gravel) moving across a channel can reduce channel width by accumulating on the updrift side or it can be deposited along the bottom of the channel, reducing channel depth (Figure 1). After equilibration, the side slope of an entrance channel dredged in open water on a sandy shore will typically range between 8 and 10 deg (Buonaiuto, Kraus, and Bokuniewicz 2000), so that there is considerable vertical exaggeration in Figure 1.

Sediment can pass over the channel by moving in suspension, and material deposited in the channel can be resuspended and transported out. A channel traps sand arriving to it from either side and, if the material remains within the channel, gives a measure of the gross longshore transport rate along a coast. Sand entering a channel may be transported seaward during ebb-tidal flows, and into the bay during flood-tidal cycles. For channels with riverine sources, shoaling may also result by deposition of upland sediments. Bypassing (sand moving over, through, or around the channel) can occur to either side as well, and the bypassing rate in the predominant (net) direction to the downdrift beach is typically required in coastal inlet projects.

In Figure 1, h_p is the authorized project depth, the minimum allowable channel depth. The authorized project depth, authorized width at the bottom, and authorized side slopes define the channel cross section. Allowable dredging tolerances for the bottom (called overdepth, typically 0.3 to 0.6 m (1 to 2 ft) and for the side slopes account for dredging inaccuracies and define the allowable pay cross section of the channel.

Estimates are made to determine the increase in dredging and cost that will accompany channel deepening and/or widening. The deepening and widening may be done under existing authority or in response to a change in authorization. In addition, deepening and, sometimes, widening may be done under a plan of advance maintenance dredging. In advance maintenance dredging, a certain length of channel is deepened (and, possibly, widened) to reduce dredging frequency.

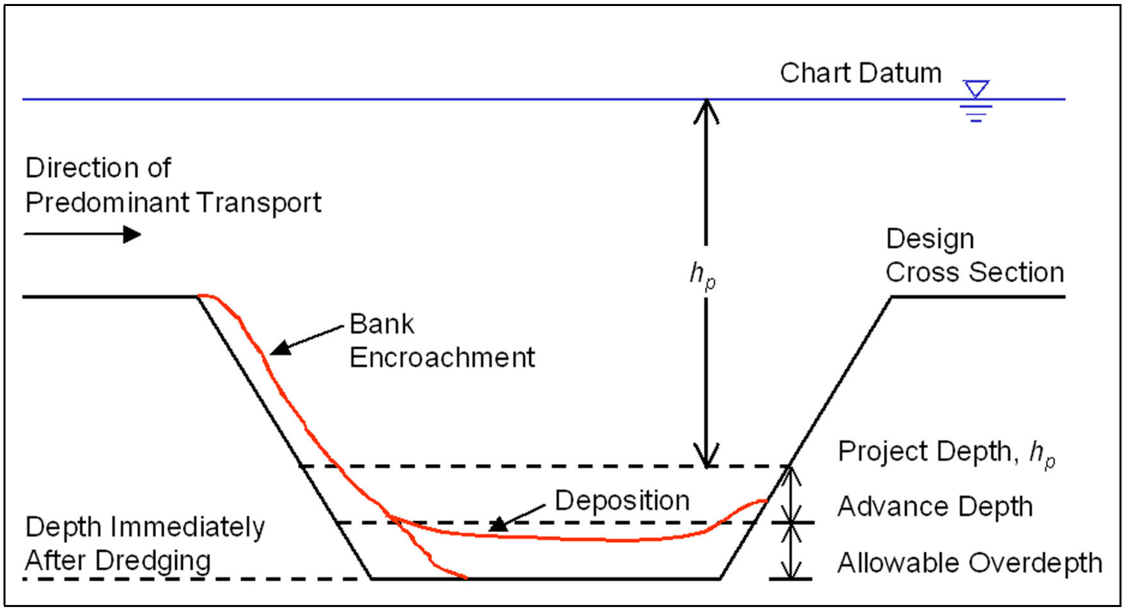

Figure 1. Definition sketch for terminology

Advance maintenance can yield cost savings by reducing the number of mobilizations, demobilizations, and surveys, or by dredging when equipment can be shared between or among projects. Also, advance maintenance may be considered to take advantage of favorable weather windows, either to reduce the cost of dredging or to maintain the channel through seasons when maintenance is not feasible.

A standard empirical methodology for estimating channel infilling is based on regression analysis of data on either dredged volume or on soundings at the study site or at channel with similar conditions (Trawle 1981). The empirical procedure requires substantial data and time for analysis. An analytical approach was given by Galvin (1982) and has been applied to Federal dredging projects (Foreman and Vallianos 1984).

This Technical Note presents an analytic method for estimating the time-evolution of sand depositing in and bypassing a channel of specified characteristics exposed to an active zone of longshore or other known cross-channel sediment transport. It is applicable to channels in estuaries, bays, and lakes by input of the rate of transport approaching normal to the channel, if the size diameter of the material is in the sand and gravel range. Infilling by cohesive sediment is not considered here. The model can be applied to estimate necessary depth and width of a channel to be newly dredged or the performance of a channel to be deepened and widened. The model is based on the continuity equation governing conservation of sand, together with typically available or estimated input transport rates. Although predictions by this method can be made quickly, they should be examined in light of experience with shoaling of the same or other channels of similar characteristics. The model is compatible with other predictive techniques being developed in the Coastal Inlets Research Program (CIRP), and it is anticipated that it will be further generalized and combined with other techniques.

CALCULATION METHOD: The channel infilling can occur through an arbitrary combination of bed-load transport, decreasing the channel width, and suspended-load transport (decreasing the channel depth). Bypassing of the channel is represented by suspended load passing over the channel and by resuspension and transport of material that has been deposited in the channel. Assumptions underlying the model are as follows:

a. Infilling by bed load can create a shoal at the edge of the channel and thereby constrict the channel (bank encroachment). The encroachment decreases the width of the channel.

b. Sediment can be deposited directly into the channel.

c. The slope of the channel remains constant. (After dredging, in particular, slumping may occur to achieve the angle of repose, and this process is neglected.)

d. The channel does not erode on the downdrift side.

e. Channel slopes are sufficiently mild that flow separation and secondary circulation do not occur or can be neglected.

f. Sediment transport along the channel, as by tidal action or a river current, is negligible. (Transport by ebb and flood currents along the channel will be introduced in a future version of the model.)

g. The cross-channel (longshore) transport is predominantly unidirectional. (This assumption can be eliminated in numerical solution of the model.)

h. Material that is deposited in the channel can be resuspended and leave the channel, and the rate of resuspension is proportional to the depth in the channel and the rate of deposition.

Figure 2 illustrates the conceptual framework of the model for the situation of transport directed to the right, assumed to be the dominant direction of transport. A general version of the model can treat both left- and right-directed transport. Immediately after dredging, the channel has width W_0 and depth h_0. The ambient or natural depth in the vicinity of the channel is h_a. As sediment is transported to the channel, it can become narrower by filling from the side and shallower by filling from the bottom. The coordinate z measures elevation from the bottom of the dredged channel. It is convenient to work with elevation from the dredged bottom rather than depth; conversion to depth below the navigation datum can then be made through knowledge of z, h_0, and h_p.

If the channel becomes narrower because of growth of the updrift side by bed-load transport and deposition into the channel, the width of the channel at a given time is

$$W(x,t) = W_0 - x(t), \qquad \text{for } x < W_0 \tag{1}$$

The transport rate q_R per unit length of channel near the updrift side of the channel can be divided into the bed-load transport rate q_{bR}, the rate q_{dR} of suspended material deposited into the channel, and the rate q_{sR} of suspended material passing over the channel from the right. For that portion of channel crossing the surf zone, the transport rate per unit length at the channel can be estimated as the total transport rate Q multiplied by the ratio of length of channel exposed to the longshore transport to the total width of the surf zone.

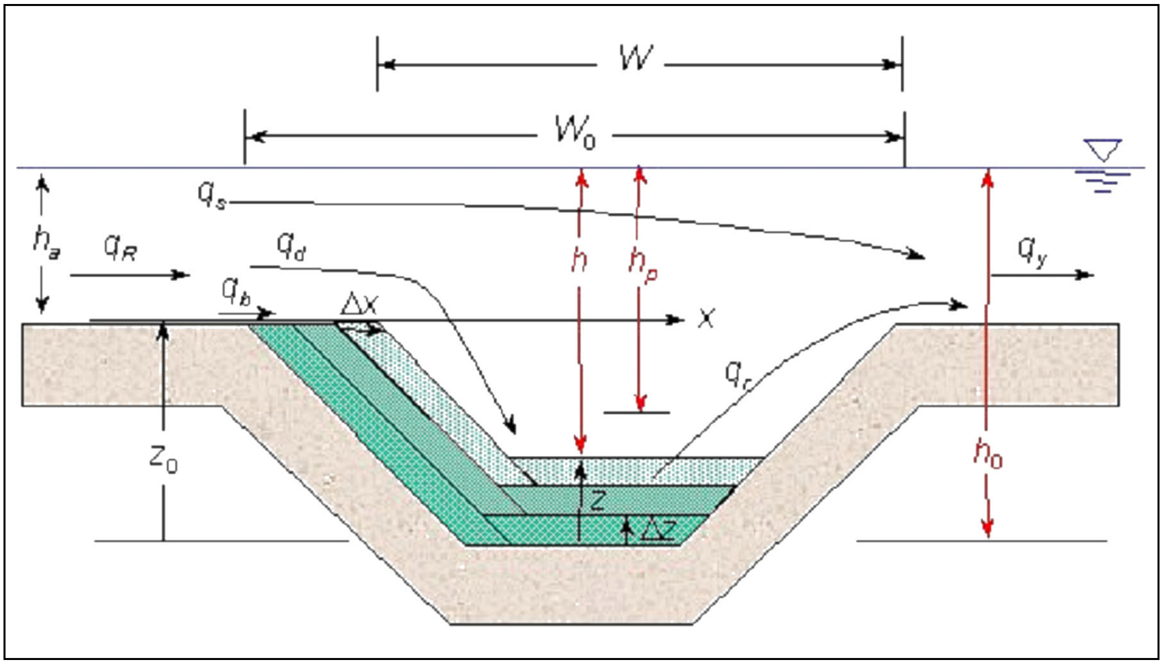

Figure 2. Definition sketch for channel infilling model

From Figure 2, the transport rate per unit length of channel is apportioned as

$$q_R = q_{bR} + q_{dR} + q_{sR} \tag{2}$$

The rate q_{cR} of material filling the channel is the sum of that entering and that resuspended and leaving the channel,

$$q_{cR} = q_{bR} + q_{dR} - q_{rR} \tag{3}$$

The rate q_{yR} of material bypassing the channel is given by the sum of q_{sR} and the rate q_{rR} of material resuspended from the channel,

$$q_{yR} = q_{sR} + q_{rR} \tag{4}$$

It is seen that $q_R = q_{cR} + q_{yR}$ $(= q_{bR} + q_{dR} + q_{sR})$.

A closure assumption of the model is that the rate of material resuspended from the channel bottom is proportional to the product of the depth of the channel and the rate of deposition as

$$q_{rR} = \varepsilon_{rd} \frac{z}{z_a} q_{dR} \tag{5}$$

where ε_{rd} is an efficiency factor for resuspension such that $0 \le \varepsilon_{rd} \le 1$. For example, for fine- and medium sand, it is expected that $\varepsilon_{rd} \cong 1$, whereas for coarse material such as gravel, $\varepsilon_{rd} \cong 0$.

To proceed, the apportionment of q_R must be known. For this purpose, coupling coefficients $a_{\alpha\beta}$ are introduced, where the a's are numbers (which can be expressed as percentages), and subscripts denote coupling between rate α and rate β:

$$\begin{aligned} q_{bR} &= a_{bR} q_R \\ q_{dR} &= a_{dR} q_R \\ q_{sR} &= a_{sR} q_R \end{aligned} \tag{6}$$

These coefficients obey the constraint

$$a_{bR} + a_{dR} + a_{sR} = 1 \tag{7}$$

The constraint expresses one equation in three unknowns, requiring two additional equations. To proceed, in the absence of process-based estimates, one can, for example, specify a_{bR} and a_{dR} as inputs and solve for a_{sR} as $a_{sR} = 1 - a_{bR} - a_{dR}$. The determination of the coupling coefficients, which should be time dependent, in terms of the coastal processes at the site is the subject of future work. At the moment, values are specified based on experience gained with the model (see the examples that follow). An estimate for the coupling coefficient a_{dR} is given in CHETN-IV-34 (Larson and Kraus 2001), called the "trapping ratio" or p in that Technical Note.

For the channel bottom, the continuity equation gives a change in bottom elevation Δz in time interval Δt as

$$W \Delta z = \left(q_{dR} - q_{rR} \right) \Delta t = \left(q_{dR} - \varepsilon_{rd} \frac{z}{z_a} q_{dR} \right) \Delta t = a_{dR} q_R \left(1 - \varepsilon_{rd} \frac{z}{z_a} \right) \Delta t$$

which becomes

$$\frac{dz}{dt} = \frac{a_{dR}}{W_0 - x} q_R \left(1 - \varepsilon_{rd} \frac{z}{z_a} \right), \qquad z(0) = 0 \tag{8}$$

Similarly, for infilling by growth of the side channel, continuity gives

$$\Delta x \left(z_a - z \right) = q_{bR} \Delta t = a_{bR} q_R \Delta t$$

which becomes

$$\frac{dx}{dt} = \frac{a_{bR}}{z_a - z} q_R, \qquad x(0) = 0 \tag{9}$$

Equations 8 and 9 are simultaneous nonlinear equations for channel depth z and width x as a function of the input rate (which can be time dependent) and time. Equation 8 indicates that z will increase more rapidly as the width decreases, and Equation 9 indicates that the width $W(x)$ will increase more rapidly as the channel fills. These equations can be solved numerically for a general situation with time-dependent variables. An analytic solution approach for rapid desk study is given next.

ANALYTICAL SOLUTION FOR CHANNEL INFILLING: Operation and maintenance of channels will not allow the depth to become less than project depth or allow the width of the channel to be greatly reduced. These conditions are equivalent to stating that interest concerns a relatively short time interval after dredging as compared to the total time it would require to fill the channel completely. For this case, the equations can be linearized under the assumptions $z/z_0 << 1$ and $x/W_0 << 1$. By expansion of denominators, Equations 8 and 9 become

$$\frac{dz}{dt} = \frac{a_{dR}}{W_0} q_R \left(1 - \varepsilon_{rd} \frac{z}{z_a} + \frac{x}{W_0}\right) \qquad z(0) = 0 \tag{10}$$

and

$$\frac{dx}{dt} = \frac{a_{bR}}{z_a} q_R \left(1 + \frac{z}{z_a}\right) \qquad x(0) = 0 \tag{11}$$

which are now simultaneous linear equations for z and x.

Differentiating Equation 10 with respect to time and substituting Equation 11 into the resultant equation to replace the dx/dt gives

$$\frac{d^2 z}{dt^2} + 2b\frac{dz}{dt} - cz = d, \qquad z(0) = 0, \quad z'(0) = \frac{a_{dR}}{W_0} q_R \tag{12}$$

where the quantities b, c, and d are

$$b = \frac{\varepsilon_{rd}\, a_{dR}}{2\, W_0 z_0} q_R, \qquad c = \frac{\varepsilon_{rd}\, a_{bR} a_{dR}}{W_0^2 z_0^2} q_R^2, \qquad d = cz_0 \tag{13}$$

A second initial condition for z was introduced through the first derivative as determined from Equations 8 evaluated with the initial conditions on x and z. The solution of (12) is found to be

$$z = C_1 \exp(r_1 t) + C_2 \exp(r_2 t) - z_0 \tag{14}$$

where

$$r_1 = -b + \sqrt{b^2 + c}, \qquad r_2 = -b - \sqrt{b^2 + c} \tag{15}$$

and

$$C_1 = \frac{z'(0) - r_2 z_0}{r_1 - r_2}, \qquad C_2 = -C_1 + z_0 \tag{16}$$

It can be seen from Equations 14 and 15 that this solution is valid for relatively short times after $t = 0$ because the term proportional to $\exp(r_1 t)$ diverges for long elapsed time ($r_1 > 0$).

Substituting (14) into (9) and integrating gives

$$x = \frac{a_{bR}}{z_0^2} q_R \left[\frac{C_1}{r_1}\left(\exp(r_1 t) - 1\right) + \frac{C_2}{r_2}\left(\exp(r_2 t) - 1\right) \right] \tag{17}$$

For small t, (14) can be expanded to give (retaining leading order in t, a_{dR}, and a_{bR}),

$$z = \frac{\varepsilon_{rd}\, a_{dR}}{W_0} q_R t - \frac{\varepsilon_{rd}\, a_{dR}}{2W_0^2 z_0}\left(a_{dR} + a_{bR}\right) q_R^2 t^2 \tag{18}$$

indicating that the channel starts filling linearly with time. If $a_{bR} = 0$ (no bed-load transport), then $x = 0$ for all time, and Equation 18 reduces to

$$z = z_0 \left[1 - \exp\left(\frac{-\varepsilon_{rd}\, a_{dR}}{W_0 z_0} q_R t \right) \right] \tag{19}$$

which indicates exponential filling of the channel.

Similarly, for small t, Equation 17 yields

$$x = \frac{a_{bR}}{z_0} q_R t + \frac{\varepsilon_{rd}\, a_{bR}\, a_{dR}}{2 W_0 z_0^2} q_R^2 t^2 \tag{20}$$

showing that if $a_{dR} = 0$ (no suspended sediment), the channel fills in linear manner with time by growth and intrusion of the updrift side into the channel.

Channel Infilling Rate: The rate of channel infilling, how rate at which the bottom is shoaling, is $R_z = dz/dt$ and can be from Equation 14. For a time shortly after dredging, Equation 18 gives

$$R_z = \frac{a_{dR}\varepsilon_{rd}}{W_0} q_R - \frac{a_{dR}\varepsilon_{rd}}{W_0^2 z_0}\left(a_{dR} + a_{bR}\right) q_R^2 t \tag{21}$$

The leading-order term is independent of z_0, so the rate of channel infilling depends more strongly on W_0 than on z_0. The solution thus indicates that the rate of channel infilling can be reduced more by increasing channel width than by increasing channel depth.

Bypassing Rate: The bypassing rate is given as $q_{yR} = q_{sR} + q_{rR}$ or

$$q_{yR} = \left(1 + a_{dR} - \varepsilon_d a_{dR} \frac{z}{z_0}\right) q_R \tag{22}$$

obtained as a function of time from Equation 14 for z.

Time Interval for Maintenance Dredging: A channel section is dredged to a design depth including a certain amount of advance dredging and a certain amount of allowable overdredging. The analytical channel infilling model provides an estimate of the maximum possible time interval Δt_p between dredging (the dredging cycle) for an assumed constant rate of infilling. Then for an increase in channel elevation from initial depth h_0 (elevation $z = 0$) to some the project depth h_p (or elevation $z_p = h_0$ - h_p), at which time dredging must be scheduled, can be determined from Equation 14 by iteration.

If bed-load transport and channel bank encroachment are not significant, then Equation 19 can be solved to give

$$\Delta t_p = -\frac{W_0 z_0}{\varepsilon_{dR} a_{dR} q_R} \ln\left(1 - \frac{z_p}{z_0}\right) = -\frac{W_0 (h_0 - h_a)}{\varepsilon_{dR} a_{dR} q_R} \ln\left(\frac{h_p - h_a}{h_0 - h_a}\right) \tag{23}$$

This equation indicates that the time between dredging intervals is directly proportional to the width of the channel; approximately proportional to the initial depth of the channel with respect to the ambient depth; and inversely proportional to the input transport rate. If Equation (23) is expanded or, equivalently, Equation 18 is solved for Δt_p to leading order, the result is

$$\Delta t_p \cong \frac{W_0 z_0}{\varepsilon_{dR} a_{dR} q_R}\left(h_0 - h_p\right) \tag{24}$$

EXAMPLE SOLUTIONS: In these two examples, z_0 = 4 m, W_0 = 50 m, and Δt = 0.1 year. The effective channel length, determined as the average width of the surf zone over all tides and wave conditions, was estimated to be 1,000 m.

Equations 8 and 9 (simultaneous nonlinear equations) were solved numerically, and the analytical model developed from the linearization (Equations 14 and 17) was also run. The simulation time was 2 years, and the numerical calculation was halted if z reached z_0/2 (assumed project depth) or x reached W_0/2 (minimum allowable width of channel).

Example 1: (fine sand) Q_R = 150,000 m^3/year, a_{dR} = 0.5, a_{bR} = 0.1, ε_{rd} = 1

The sediment at this site is fine sand, and calculation of the trapping factor according to Larson and Kraus (2001) gave **,** a_{dR} = 0.5. This example simulates a shallow-draft channel at an inlet located on a sandy shore, so most of the fine sand is deposited into the channel or passes over the channel ($a_{sR} = 1 - a_{dR} - a_{bR} = 0.4$). If material is deposited into the channel, it can be readily resuspended. The effective channel length is 1,000 m, so q_R = 150,000/1,000 = 150 m^3/m/year.

Figures 3 and 4 compare calculations with the numerical model and the linearized model. For short elapsed time there is agreement, with deviations occurring after about 0.6 to 0.8 year for this example. After 1.7 years, project depth ($z = z_a$) was reached, and the numerical model stopped.

Figure 5 shows the time evolution of the channel infilling rate q_{cR} and the bypassing rate q_{bR}, normalized by the input q_R. The total adds to unity at any given time. The rate of bypassing exceeds the channel infilling rate approximately 0.6 years after dredging.

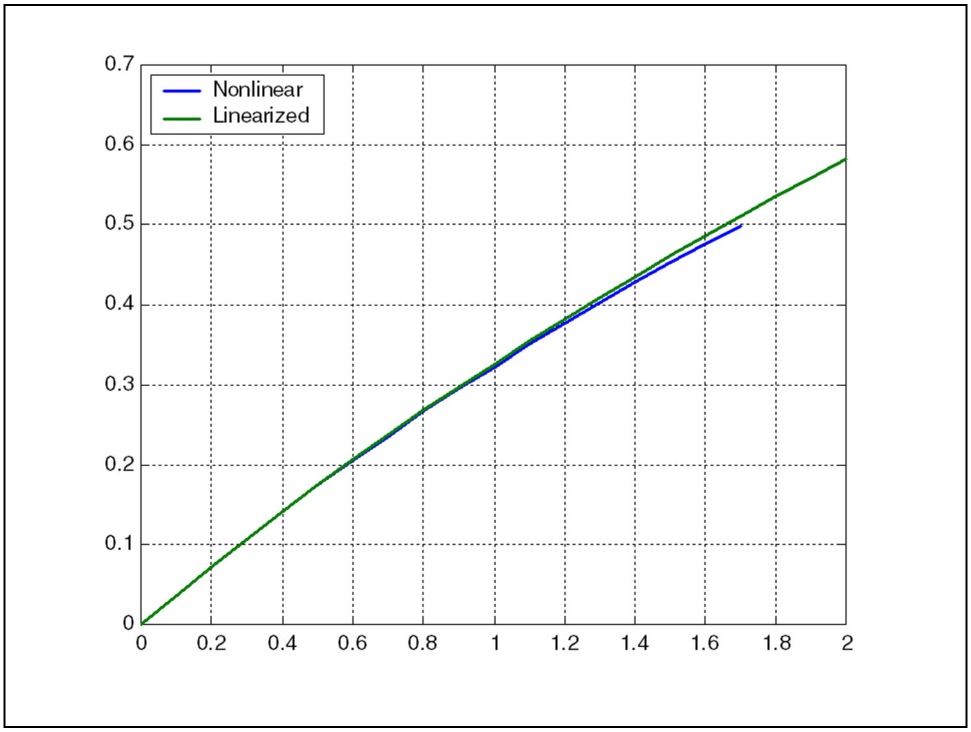

Figure 3. Increase in elevation (decrease in depth) in channel on sand shore: comparison of nonlinear model (numerical solution) and linearized model (analytical solution)

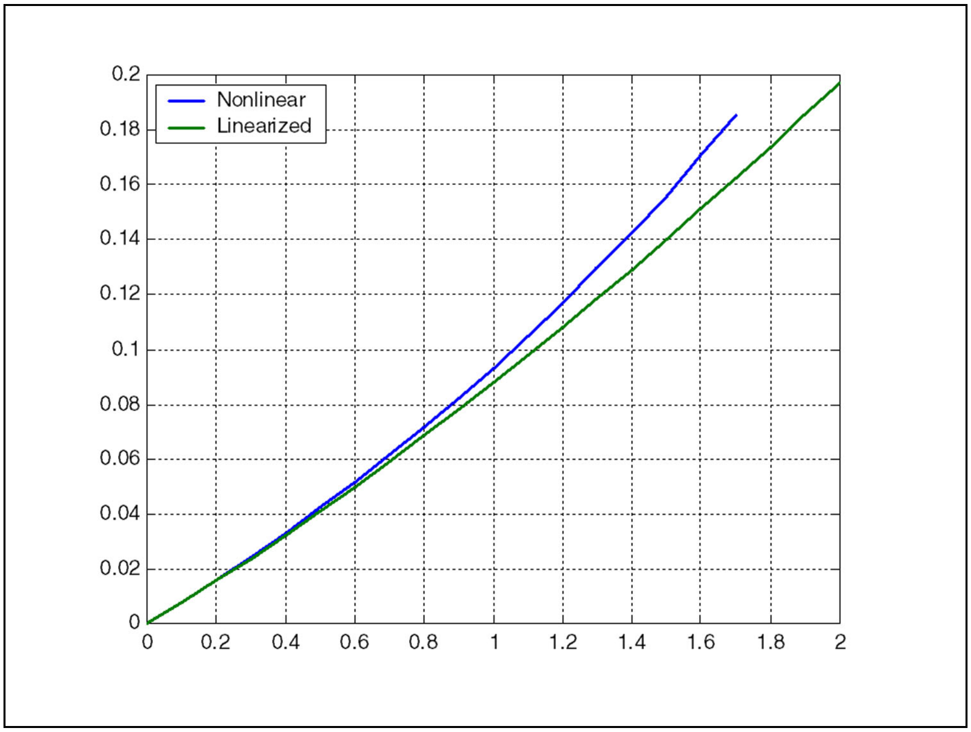

Figure 4. Increase in intrusion distance (decrease in width) in channel on sand shore: comparison of nonlinear model (numerical solution) and linearized model (analytical solution)

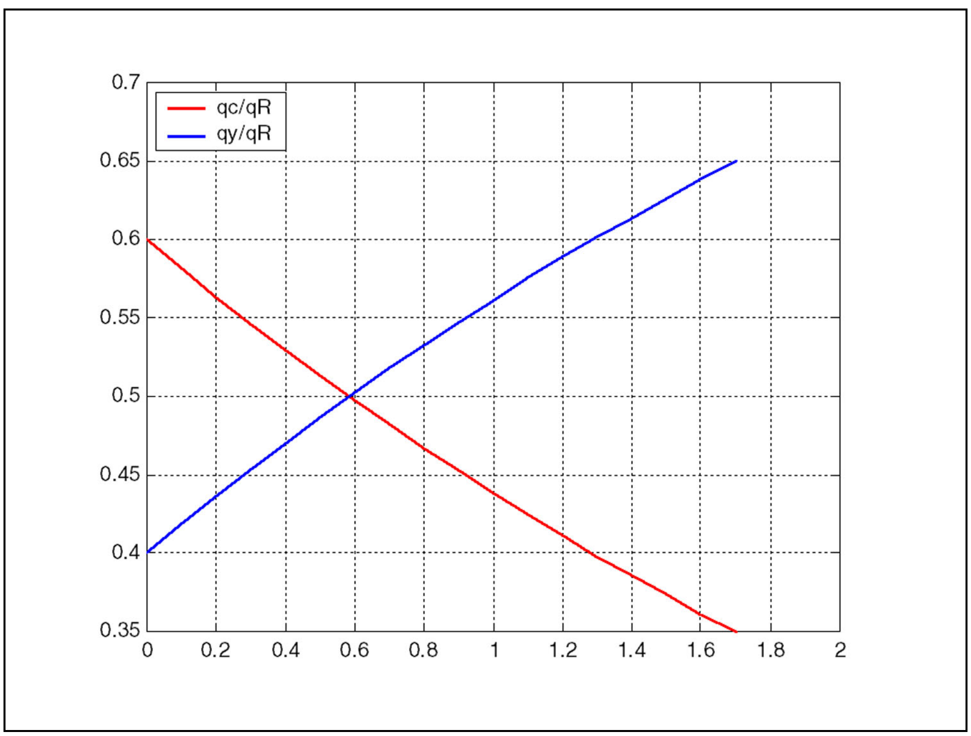

Figure 5. Evolution of channel infilling rate and channel bypassing rate on sand shore

Example 2: (gravel) Q_R = 50,000 m^3/year, a_{dR} = 0.2, a_{bR} = 0.7, ε_{rd} = 0

This example simulates a shallow-draft channel in an inlet located on a gravel shore, so most of the coarse-grained material is deposited on the updrift side of the channel, with little bypassing by suspended transport ($a_{sR} = 1 - a_{dR} - a_{bR} = 0.1$). Only the fine material is assumed to travel over the channel by suspension. If the gravel gets into the channel, none is resuspended sufficiently to leave it. The effective channel length is 1,000 m, so q_R = 50,000/1,000 = 50 m^3/m/year.

In this situation of a gravel shore, because most material remains at the updrift side of the channel, little depth is lost (Figure 5). However, after 2 years, the updrift side of the channel has intruded about 37 percent of the way across the channel (Figure 6), becoming a hazard to navigation. The side of the channel grows approximately linearly, because little material is deposited in the channel bottom through suspension. Therefore, the governing equation is only weakly nonlinear, and the linearized (analytical) solution and numerical solution produce almost the same results. Figure 7 shows that most of the material is deposited into the channel, with little bypassing.

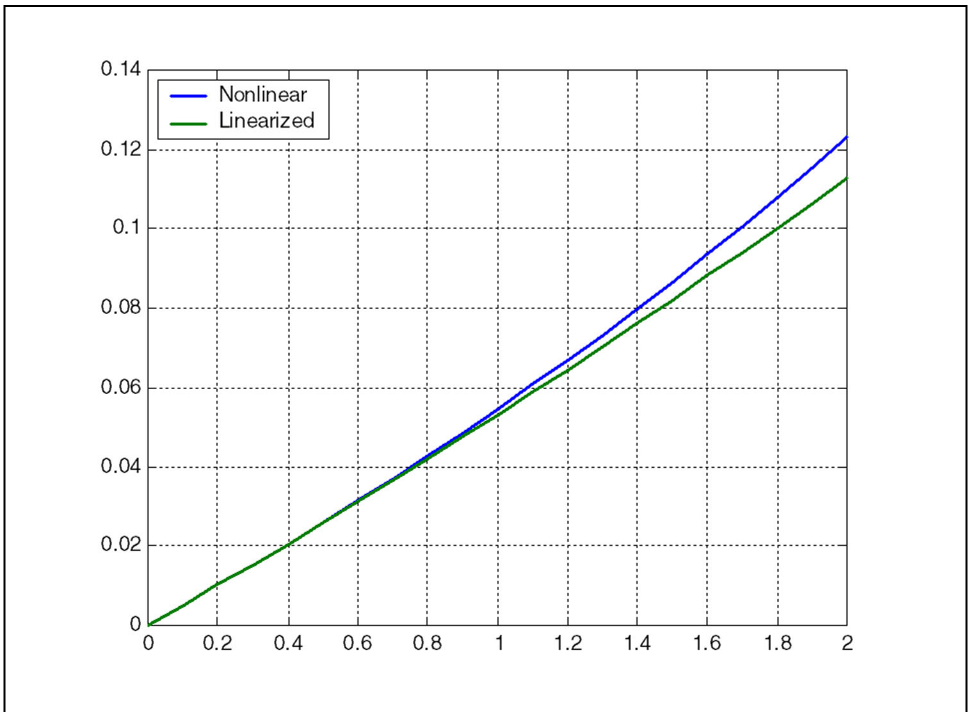

Figure 6. Increase in elevation (decrease in depth) in channel on gravel shore: comparison of nonlinear model (numerical solution) and linearized model (analytical solution)

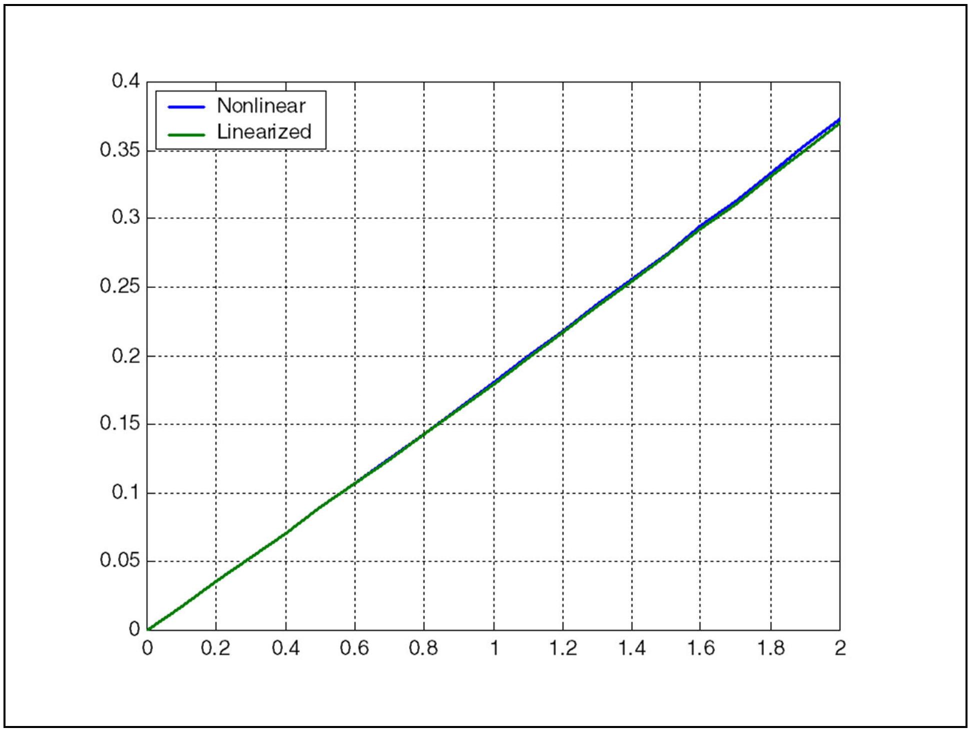

Figure 7. Increase in intrusion distance (decrease in width) in channel on gravel shore: comparison of nonlinear model (numerical solution) and linearized model (analytical solution)

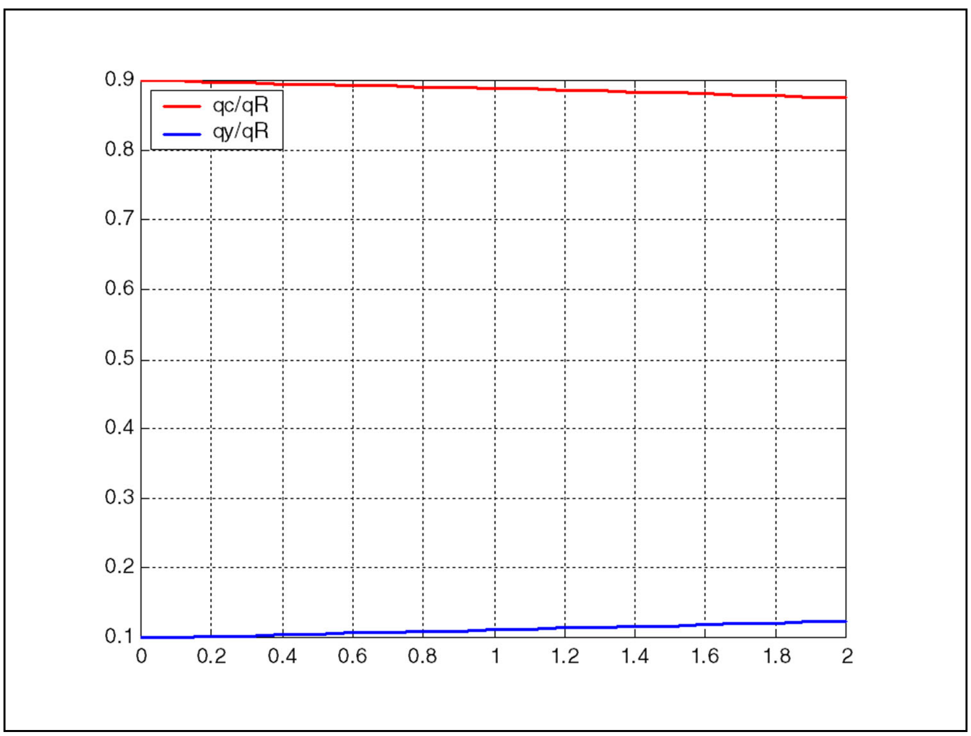

Figure 8. Evolution of channel infilling rate and channel bypassing rate on gravel shore

FUTURE WORK: Research is underway in the CIRP to provide a convenient interface for implementing the numerical solution of the channel infilling model. The solution will allow time-dependent wave information to generate a longshore current, calculate the width of the surf zone, and channel infilling by sections with different ambient depths along the channel.

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