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Estimating Along-Channel Flow and Sediment Transport at Tidal Entrances — EbbJet Calculator

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PURPOSE: The Coastal Engineering Technical Note (CHETN) herein describes the EbbJet Calculator, a utility of the Analytical Toolbox of the Diagnostic Modeling System (DMS) (Kraus 2000). The EbbJet Calculator estimates depth-averaged velocity and sediment transport rate at an inlet from established analytical and empirical equations.

BACKGROUND: As described in CHETN IV-28 (Kraus 2000), "Introduction to the Diagnostic Modeling System (DMS)," the DMS is an experienced-based methodology for addressing sediment-shoaling problems. It comprises a suite of diagnostic tools intended for planning, design, and maintenance of navigation projects. The DMS methodology provides engineering guides, procedures, and analytical tools founded upon the principles of fluid dynamics and sediment transport. As a diagnostic procedure, the DMS is not intended to replace the level of detailed information required for final design of newly authorized channels or for in-depth study of an existing navigation project. Rather, it is intended to function as a screening tool to quickly assess existing shoaling problems, to categorize them according to their key characteristics, and to identify corrective actions permissible within the project's existing authorization.

The DMS has three components—the Manual, the Data Manager (Johnston and Craig 2002), and the Analytical Toolbox. The Analytical Toolbox is a collection of calculation tools and guidance documents assembled for understanding and quantifying shoaling conditions and for formulating and evaluating alternatives to reduce dredging. The calculation tools consist primarily of analytical and simple numerical models for estimating hydrodynamics and sediment transport in and around channels for a variety of situations. These tools are easy-to-use, stand-alone applications.

The EbbJet Calculator estimates tidal current velocity along an entrance channel of an inlet based on the theory of Ozsoy and Ünlüata (1982). In their theory, the velocity distribution of an ebb- or flood-tidal jet is calculated including lateral mixing and entrainment, bottom friction, one-dimensional bathymetric changes, and ambient currents. The EbbJet Calculator also provides estimates of sediment transport through equations developed by van Rijn (1984a, 1984b, and 1984c). Knowledge of the flow patterns and sediment transport around inlets will aid in optimizing channel alignment and estimating shoaling rates under possible planned modifications.

THEORY: The EbbJet program employs the theory of Ozsoy and Ünlüata (1982) to simulate tidal inlet hydraulics on ebb tide.[1] Analytic equations are developed from the depth-averaged equations of motion taking into account lateral mixing and entrainment, bottom friction, and one-dimensional bathymetry change. The EbbJet program does not take into account the ambient current. The theory is based on the assumption of an idealized inlet with equal shoreline offsets. The offshore bathymetry consists of straight and parallel contours. The Darcy-Weisbach friction factor is used to model bottom shear stress and remains constant over the solution domain.

Consider the turbulent jet flow produced by water ebbing from an inlet into a semiinfinite ocean as depicted in Figure 1. In the figure, x is the offshore distance, y is the longshore distance from the center of the inlet, b(x) is the half-width of the jet, r(x) is the half-width of the jet core (region where the velocity is the same as the inlet entrance), u(x,y) is the x component of velocity at any point in the jet, u_c(x) is the x-component of velocity along the jet center line, and h(x) is the depth (from still-water conditions). The constants h_0, b_0, and u_0 describe the depth at the inlet, the half-width of the inlet, and the velocity at the inlet.

The reduced equations of motion (conservation of mass and momentum) take the following form:

$$\frac{\partial hu^2}{\partial \mathrm{x}} + \frac{\partial huv}{\partial \mathrm{y}} = -\frac{f}{8}u^2 + \frac{1}{\rho}\frac{\partial}{\partial \mathrm{y}}F_{\mathrm{xy}}$$

$$\frac{\partial hu}{\partial \mathrm{x}} + \frac{\partial hv}{\partial \mathrm{y}} = 0 \tag{1}$$

where h is the local water depth, u and v are the x- and y- directed velocities, respectively, f is the Darcy-Weisbach friction factor, ρ is the density of water, and F_{xy} is the cross coupling term of the depth-integrated stress tensor (including turbulent fluctuations).

To solve these equations, the velocity distribution is assumed to be self-similar with respect to the nondimensional coordinate:

$$\zeta = \frac{|\mathrm{y}|}{b(\mathrm{x})} \tag{2}$$

The similarity profile is adopted from Abramovich (1963). It has the form

$$\frac{u(\mathrm{x,y})}{u_c(\mathrm{x})} = J(\xi) \tag{3}$$

[1] The EbbJet Calculator is also capable of estimating the current velocity and potential sediment transport for the flood current. For the purposes of this CHETN, only calculation of the ebb current is described.

where

$$J(\xi)=\begin{cases} 0 & ; \quad 1<\bar{\xi} \\ (1-\bar{\xi}^{1.5}); & 0<\bar{\xi}<1 \\ 1 & ; \quad \bar{\xi}<0 \end{cases} \qquad \bar{\xi}=\frac{\xi - r/b}{1-r/b} \tag{4}$$

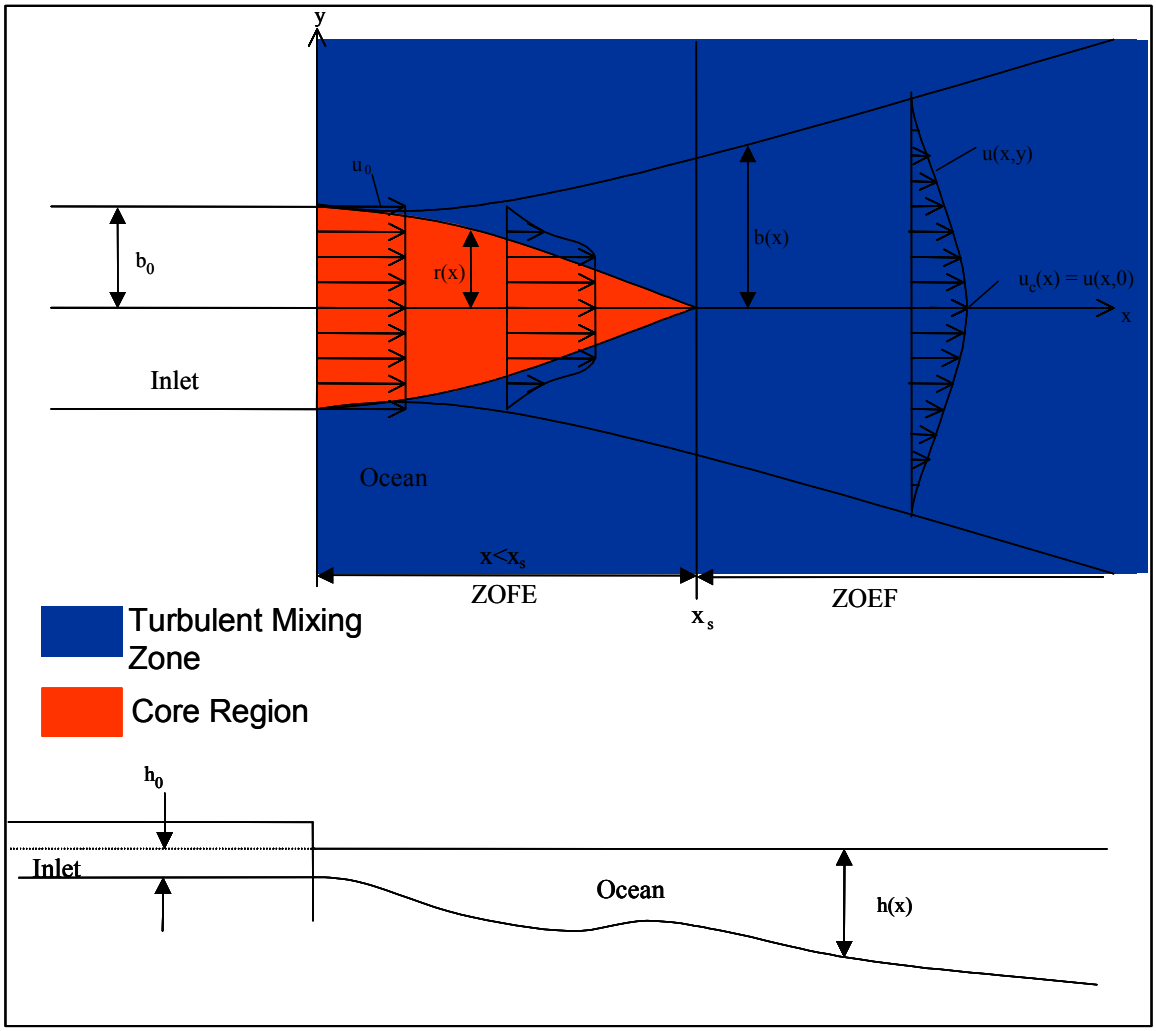

Figure 1. Inlet jet definition sketch

In addition, following classical jet theory, two zones are delineated within the flow field:

Zone of Flow Establishment (ZOFE) — $0< x < x_s$
Zone of Established Flow (ZOEF) — $x > x_s$

Two regions within the jet cross section characterize the ZOFE: a core region where the velocity is the same as the velocity issuing from the orifice (inlet, x = 0) and a turbulent mixing region where the velocity is less than that along the jet center line. The turbulent mixing zone, having zero width at the jet orifice, grows to cover the entire jet at the end of the ZOFE. Conversely, the core region extends through the entire jet cross section at the orifice and reduces to zero at the

end of the ZOFE. The ZOEF is characterized by a turbulent mixing region that extends throughout the jet cross section. In the ZOEF, the jet is fully developed and the core region has a width of zero.

Characteristics of each of these regions allow for simplification of the differential equations to an analytical solution given the following boundary conditions:

$$\begin{aligned} b(0) &= b_0 \\ r(0) &= b_0 \\ u_c(0) &= u_0 \\ r(\mathrm{x}) &= 0 \quad ; \quad \mathrm{x} > \mathrm{x}_s \\ u_c(\mathrm{x}) &= u_0 \quad ; \quad \mathrm{x} < \mathrm{x}_s \\ v\left[\mathrm{x}, b(\mathrm{x})\right] &= a u_c(\mathrm{x}) \end{aligned} \tag{5}$$

The final condition states that the longshore velocity (entrainment velocity) at the jet's edge is proportional to the velocity along the jet center line through the constant, *a*, where *a* is equal to 0.036 in the ZOFE and 0.050 in the ZOEF (empirically determined).

Defining the following dimensionless parameter allows for reduction of the equations of motion:

$$\xi = \frac{\mathrm{x}}{b_0}, \quad \mu = \frac{f b_0}{8 h_0}, \quad H(\xi) = \frac{h_0}{h_0}, \quad R(\xi) = \frac{r}{b_0}, \quad B(\xi) = \frac{b}{b_0}, \quad U(\xi) = \frac{u_c}{u} \tag{6}$$

The equations of motion are then solved to yield the following results for the ZOFE:

$$U = 1, \quad R = \frac{0.45F - 0.314G}{0.134H}, \quad B = \frac{0.683G - 0.55F}{0.134H} \tag{7}$$

where

$$F = \exp\left(-\mu \int_0^{\xi} \frac{d\xi'}{H(\xi')} \right), \quad G = a \int_0^{\xi} H(\xi') d\xi' + 1 \tag{8}$$

In the ZOEF, the following equations apply:

$$R = 0, \quad U = \frac{F}{\sqrt{L}}, \quad B = \frac{L}{0.316HF} \tag{9}$$

where

$$L = \frac{0.632a}{0.45} \int_{\xi_s}^{\xi} H(\xi')F(\xi')d\xi' + F^2(\xi_s), \quad \xi_s = \frac{x_s}{b_0} \tag{10}$$

Both sets of equations yield solutions for b, r, and u_c (and hence u(x,y)) given the inputs of b_0, u_0, f, and h(x).

Sediment transport is estimated from the equations developed by van Rijn (1984a, 1984b, and 1984c) and the current velocities calculated within the EbbJet Calculator. The total sediment transport rate includes bed load and the suspended load. Van Rijn computed bed-load sediment transport as the product of the saltation height, the particle velocity, and the bed-load concentration. Saltation height and particle velocity, for a variety of sediments and flow parameters, is solved numerically with the equations of motion. The model calibrated with laboratory data. Measurements of bed-load transport provided empirical determination of the sediment concentrations in the boundary layer.

Before calculating the bed-load transport, the dimensionless particle size parameter and the transport stage parameter need to be defined. The particle size parameter, D_*, is found using the following equation:

$$D_* = D_{50}\left[\frac{(s-1)g}{\nu^2}\right]^{1/3} \tag{11}$$

In the equation, D_{50} is the median diameter of the bed material, s is the specific gravity of the sediment, g the acceleration of gravity, and ν is the kinematic viscosity of the fluid. The nondimensional transport stage parameter, T, is calculated via the following equation:

$$T = \frac{\left(u_*'\right)^2 - \left(u_{*,cr}\right)^2}{\left(u_{*,cr}\right)^2} \tag{12}$$

where u'_* is the bed shear velocity related to grain size (on the bed) and $u_{*,cr}$ is the critical shear velocity according to Shields' Curve. The parameter u'_* is an estimate for the average shear velocity at the upsloping part of the bed forms.

Given D_* and T, calculation of the bed-load transport, q_b, is:

$$q_b = D_{50}^{3/2}\left(0.053\frac{T^{2.1}}{D_*^{0.3}}\right)\sqrt{(s-1)g} \tag{13}$$

Bed-load transport is given in units of volume of transported material per unit time per unit width (van Rijn 1984b).

Van Rijn (1984b) computed the depth-integrated suspended sediment transport rate as the product of local concentration and flow velocity. A reference concentration is derived from the equations for bed-load transport. Measured concentrations provided the calibration for the equations.

The following equations result from the integration of the differential equation describing the vertical distribution of sediment concentration in a steady uniform current:

$$q_s = \frac{u_* c_a}{\kappa}\left(\frac{a}{h-a}\right)^{Z'}\left[\int_a^{0.5h}\left(\frac{h-z}{z}\right)^{Z'}\ln\left(\frac{z}{z_0}\right)dz + \int_{0.5h}^{h} e^{-4Z'(z/h-0.5)}\ln\left(\frac{z}{z_0}\right)dz\right] \tag{14}$$

where u_* is the friction velocity, c_a is the reference concentration, κ is von Karman's constant, a is the reference concentration elevation, h is the local water depth, Z' is a suspension parameter, z_0 is the zero velocity level above the bed, and z is the vertical coordinate (positive upwards from the bed). This equation is represented by the simpler expression:

$$q = F_s u h c_a$$

$$F_s = \frac{\left(\frac{a}{h}\right)^{Z'} - \left(\frac{a}{h}\right)^{1.2}}{\left(1-\frac{a}{h}\right)^{Z'}(1.2 - Z')} \tag{15}$$

for the range $0.03 < Z' < 3$ and $0.01 < a/h < 0.1$ with a maximum error of approximately 25 percent. In the equations, the reference concentration is defined as

$$c_a = \frac{0.035}{2.3}\frac{D_{50}}{a}\frac{T^{1.5}}{D_*^{0.3}} \tag{16}$$

To avoid errors associated with setting the reference elevation, a, too low, van Rijn (1984b) assumes a reference elevation equal to one-half the bed form height or the equivalent roughness height, if the bed form dimensions are unknown. The minimum reference elevation should be no less than 1/100th the local water depth.

Finally, the suspension parameter in the suspended load equation, Z', is defined by

$$Z' = Z + \varphi \tag{17}$$

where

$$Z = \frac{\omega_s}{\beta \kappa u_*}, \tag{18}$$

$$\varphi = 2.5\left(\frac{\omega_s}{u_*}\right)^{0.8}\left(\frac{c_a}{c_0}\right)^{0.4} \quad ; \quad 0.01 \le \frac{\omega_s}{u_*} \le 1 \tag{19}$$

$$\beta = 1 + 2\left(\frac{\omega_s}{u_*}\right)^{2} \quad ; \quad 0.1 \le \frac{\omega_s}{u_*} \le 1 \tag{20}$$

In the previous equations, ω_s is the sediment fall velocity and c_0 is the maximum volumetric bed concentration (=0.65).

EBBJET CALCULATOR INTERFACE: The EbbJet Calculator is a front- and back-end model with integrated interface developed for solving the equations as previously described. Upon starting the program, the user is presented with the console shown in Figure 2. The console consists of three areas: the menu bar, the graph display area, and the graph display option buttons. The menu bar runs the calculations for the program; the graph display area shows the results of the calculations, as either line or contour graphs; and the graph display option buttons select which data are presented and how they are presented.

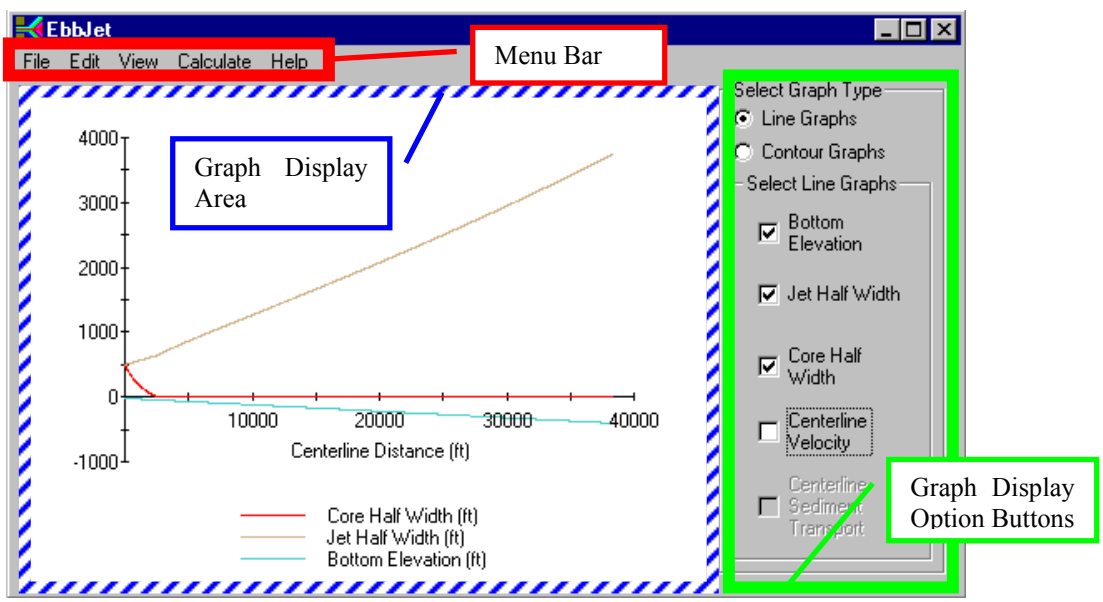

Figure 2. EbbJet console

To begin a simulation, the user can either load a simulation from an external input file or enter the information manually. To enter the information, select the menu item *Calculate|Inlet Hydraulics*. This action causes the *Inlet Hydraulics* dialog box to appear (Figure 3). Here the user can either load information from a previously saved, formatted input file or manually enter the data requested. Required data includes the depth at the inlet, the inlet half-width, the depth-averaged velocity at the inlet, the Darcy-Weisbach friction factor, a description of the offshore area (as either a constant slope or an input profile), and the units of all calculations (metric or English).

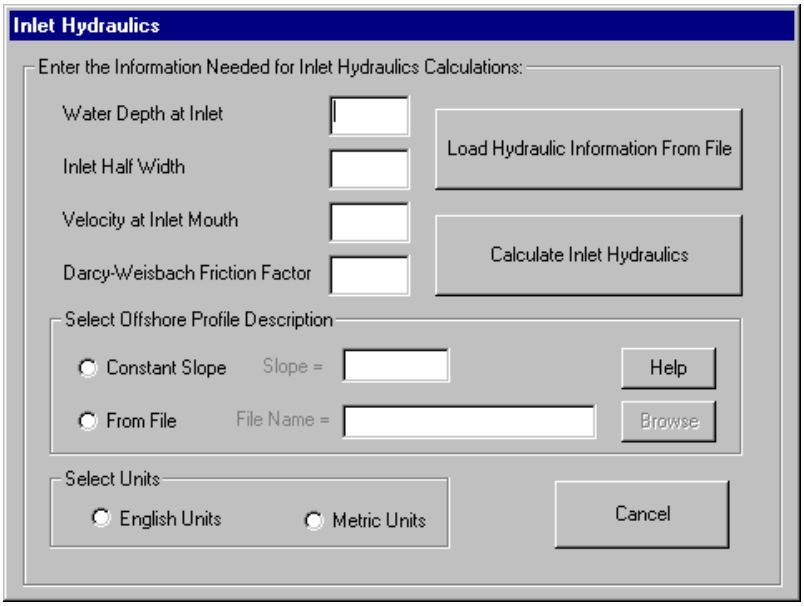

Figure 3. *Inlet Hydraulics* dialog box

After the required information has been entered, the **Calculate Inlet Hydraulics** button will solve the hydraulics equations. The program returns to the main console where the user can view the results either as line graphs (Figure 2) or as contour graphs (Figure 4). Calculation results available for display as line graphs are the input bottom elevation, the jet half-width, core half-width, center line velocity, and center line sediment transport (enabled after performing the sediment transport calculations). Contour graphs available for display are cross-shore velocity magnitude and sediment transport magnitude (enabled after performing the sediment transport calculations).

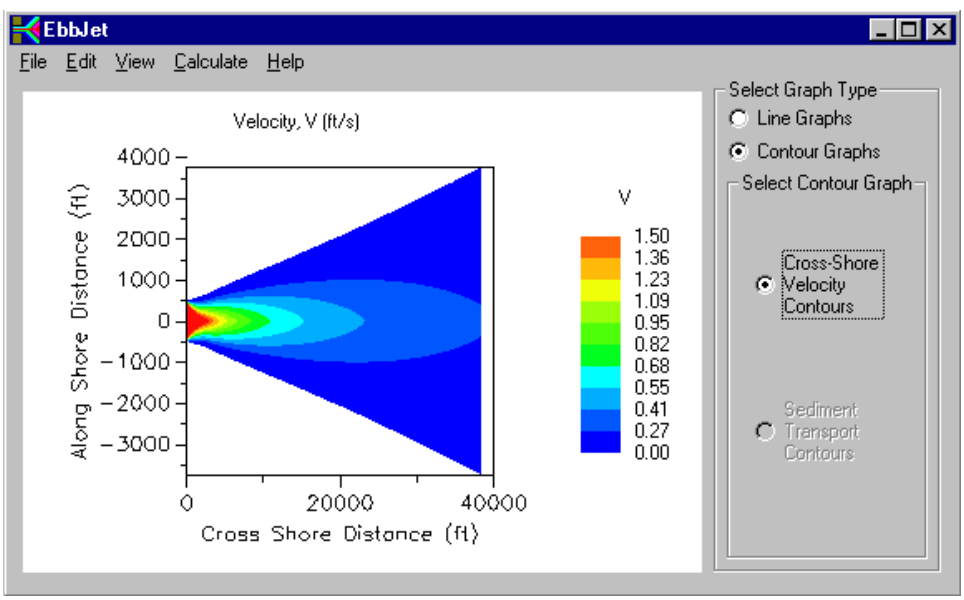

Figure 4. Contour graph of cross-shore velocity magnitude

The program is structured so that a hydraulic simulation must be completed before the sediment transport calculations can proceed. To perform sediment transport calculations, the user selects the menu option *Calculate|Inlet Sediment Transport*. This action causes the *Inlet Sediment Transport* dialog box to appear (Figure 5). Again, the user can either load the requested information from a previously saved, formatted file or enter the information manually. Required sediment information includes D_{50}, D_{90}, sediment specific gravity, and the sediment distribution's geometric standard deviation. Additionally, the fluid density and kinematic viscosity are required for the calculations. The fluid properties may be input manually or calculated from the fluid temperature and salinity.

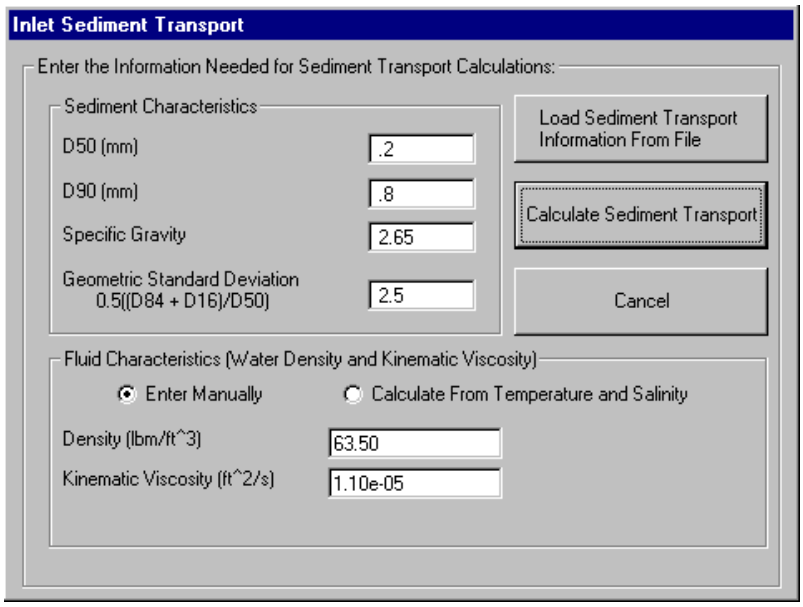

Figure 5. *Inlet Sediment Transport* dialog box

Upon entering the requested information, the user presses the **Calculate Sediment Transport** button to perform the calculations. The program then returns to the main console where the results may be displayed as either line graphs or contour graphs (Figure 6).

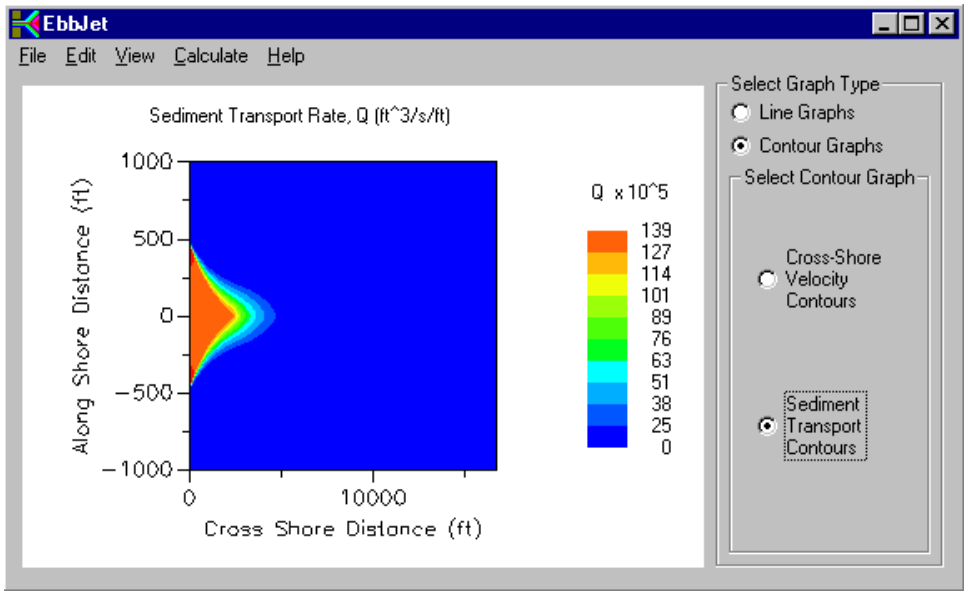

Figure 6. Contour graph of sediment transport contours

In addition to the features previously described, the EbbJet Calculator contains several other capabilities accessed through the following menu items:

a. *File|Save Simulation* — This action launches a *Save File* dialog box. The saved file contains only the inputs entered into the *Inlet Hydraulics* dialog box and the *Inlet Sediment Transport* dialog box. This file can be loaded at a later date to rerun simulations and view results.

b. *Edit|Copy Chart to Clipboard* — Selecting this option copies the visible graph (line or contour) to the clipboard. The graph is then available for pasting into any document or application as a Windows metafile.

c. *File|Export to File* — Selecting this menu item exports the temporary files used to create the graphs to ASCII files. This makes the results of the calculations available for display in an alternate software package.

d. *File|Print* — This menu item displays the *Print* dialog box. Selecting the appropriate printer and clicking **OK** will print the graph showing in the *Graph Display Area*.

e. *View|Variable Summary* — This menu option displays the *Variable Summary* window which contains all the information listed in the *Inlet Hydraulics* dialog box and the *Inlet Sediment Transport* dialog box. Text in the window may be selected and copied to the clipboard for subsequent pasting into a text editor.

f. *Help* — An extensive help file is also available through the menu bar.

Calculating the currents and sediment transport at a tidal entrance with the EbbJet Calculator provides a quick overview of the flow characteristics at the entrance. This information can help evaluate channels near inlets. For example, a hydrographic survey can be designed to adequately cover potential shoaling areas due to tidal sediment transport. By entering spring tide conditions into the EbbJet Calculator, areas of active sediment transport can be identified. The results will

provide a rough estimate of the offshore extent of the ebb tidal shoal. If the results resemble the output in Figure 6, the project manager can specify that the survey should extend approximately 1,524 m (5,000 ft) seaward of the inlet entrance.

CONCLUSIONS: The EbbJet Calculator allows quick estimation of the ebb or flood jet issuing from an inlet based on the analytical solution of flow from Ozsoy and Ünlüata (1982). Estimates of sediment transport are developed from the sediment transport equations published by van Rijn (1984a, 1984b, and 1984c). Developed for inclusion in the Diagnostic Modeling System Analytical Toolbox, the EbbJet Calculator provides estimates of flow velocity and associated sediment transport at inlets that will aid in optimizing channel location and estimation of shoaling rates.

OBTAINING SOFTWARE: The EbbJet Calculator is distributed from the DMS Web site located at http://www.taylorengineering.com/DMShome/DMSDefault.htm and at the Coastal Inlets Research Program Web site located at http://cirp.wes.army.mil/cirp/cirp.html, then go to Products and Tools.

ADDITIONAL INFORMATION: Questions about this CHETN may be addressed to Dr. Mark S. Gosselin (904-731-7040; e-mail: mark@oea-inc.com). The authors appreciate review of this CHETN by colleague Dr. R. Bruce Taylor and by Dr. Nicholas C. Kraus, Principal Investigator of the Diagnostic Modeling Work Unit, U.S. Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory. For additional information contact Dr. Julie Rosati, Julie.D.Rosati@usace.army.mil This CHETN should be cited as follows:

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