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Numerical Evaluation of Stress Intensity Factors (K_I) J-Integral Approach

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PURPOSE: The purpose of this Coastal and Hydraulics Engineering Technical Note (CHETN) is to describe the numerical evaluation of the stress intensity factors using the J-integral approach (Rice 1968a, 1968b). The stress intensity factors have been calculated for a semi-infinite plate with an edge crack. This case has a known closed-form solution, and only a 1.25 percent difference between the numerical and closed-form solution was obtained. This CHETN also presents the methodology to perform a detailed three-dimensional (3-D) meshing of any complicated geometry. The meshing approach was used to generate a 3-D mesh of a hydraulic steel structure with multiple cracks. Stress intensity factors for the 3-D problem with multiple cracks are also discussed. This approach is being developed to be used in fracture mechanics analysis of U.S. Army Corps of Engineers structures.

Steel hydraulic structures maintained by the U.S. Army Corps of Engineers have experienced numerous cases of fatigue cracking. This CHETN describes the combination of the fracture mechanics approach with the 3-D finite element analysis to calculate the stress intensity factors on cracked hydraulic steel structures. The stress intensity factors are calculated using the J-integral approach to determine the remaining capacity of the cracked sections.

BACKGROUND: Steel hydraulic structures maintained by the Corps consist primarily of lock and spillway gates, bulkheads, and other closure structures. In recent history there has been ample evidence of unsatisfactory performance of these structures. This evidence has included numerous cases of fatigue cracking caused by welded connections with low fatigue resistance, poor weld quality, unanticipated structural behavior, or unexpected loading. In the most severe cases there has been at least one catastrophic failure of a bulkhead system, and complete replacement of lock gates has been required in at least two other cases. Maintenance and repair of fatigue and fracture failures represents a major Operations and Maintenance expenditure for the Corps.

Because of these failures, analytical techniques for employing state-of-the-art capabilities for fracture mechanics analysis using finite element modeling are being developed. This ongoing effort includes the use of commercially available nonlinear finite element programs incorporating J-integral analysis for fracture analysis. The results of J-integral analysis can be directly compared to elastic or elastic-plastic material properties for assessment. These analytical tools can be used for much more accurate and detailed fracture assessments than the Corps currently has the capability to perform and can be used to access the criticality or need to perform repairs.

Detailed finite element fracture analyses employing J-integral analysis have the potential to provide a much more accurate fracture assessment. Cost savings may be realized from accurate fracture assessments through avoidance of unnecessary repairs. Detailed fracture analysis can also provide an important investigative tool in assessing the actual loads in a member at the time of failure.

This CHETN will briefly describe the linear elastic fracture mechanics (LEFM) concepts, following by a detailed description of the J-integral approach. The 3-D finite element meshing technique will also be described. The technical note will conclude with the semi-infinite plate and the 3-D hydraulic steel structure numerical evaluation of the stress intensity factors.

FRACTURE MECHANICS: Fracture mechanics provides a tool for assessing the criticality of flaws in structures. LEFM is the basic theory of fracture, originated by Griffith (1921) and completed by Irwing (1957); Rice (1968a, 1968b); and Riveros (2006). The LEFM is a highly simplified theory that is applicable to any material under a basic ideal situation: all the material is elastic except in a vanishingly small region (a point) at the crack tip (Bazant and Planas 1998).

Fracture mechanics is a method of characterizing the fracture behavior in structural parameters, stress, and flaw size that can be used directly according to Barsom and Rolfe (1987). They also state that the science of fracture mechanics can be used to describe quantitatively the trade-offs among stresses, material toughness, and flaw size.

STRESS INTENSITY FACTOR: A major achievement in the theoretical foundation of LEFM was the introduction of the stress intensity factor K (the demand) as a parameter for the intensity of stresses close to the crack tip and related to the energy release rate (Bazant and Planas 1998). Ingliss (1913) studied the unexpected failure of naval ships, and Griffith (1921) extended this work using thermodynamic criteria. Using this work, Irwing (1957) developed the concept of the stress intensity factor.

Stress intensity factors are a measure of the change in stress within the vicinity of the crack tip. Therefore, it is important to know the crack direction and when the crack stops propagating. The stress intensity factor is compared with the critical stress intensity factor K_{IC} (the capacity) to determine whether or not the crack will propagate.

Dimensional analysis can be used to show that the stress intensity factor for Mode I fracture K_I, has the following form:

$$K_I = g\sigma\sqrt{\pi a} \tag{1}$$

where

σ = nominal far field stress

$2a$ = crack length

and g is a nondimensional function depending on the size and geometry of the crack, size and geometry of the structural component, and the type of loading. For normal cracks, its value ranges between 1 and 2, but may be larger for longer cracks. Functions defined for common geometries and loading conditions are available in Barsom and Rolfe (1987) and Tada (1973).

If K_I is the same for two cracked bodies, then based on the equations, the same stress field will exist at their crack tips. If the two bodies are made of the same material, an identical response is expected. This fact leads to the important conclusion that K_I can be used as a similitude parameter to compare the response of the same material at the crack tip and also to compare the degree to which materials are influenced by the stress fields.

FRACTURE TOUGHNESS: CRITICAL STRESS INTENSITY FACTOR K_{IC}: Another important parameter of the linear elastic fracture mechanics is the fracture toughness, K_{IC} (the capacity). Fracture toughness is a material fracture property (Swamy 1979; Riveros 2006). If a plate is loaded (Figure 1) to the failure stress σ_f, and if $\sigma = \sigma_f$ is the value of the nominal far field stress at failure, then a value of K_I associated with σ_f could be determined and referred to as K_{IC}. If this toughness property is available for the material, then the failure state stress could be expressed as (Swamy 1979):

$$\sigma_f = \frac{K_{IC}}{\sqrt{\pi a}} \tag{2}$$

From Figure 1 it can be observed that when the obtained value of σ_f exceeds the tensile strength of the material f_t (or when $a < a_c$, the critical crack length), for small crack length, the LEFM is not valid. The relation between the energy release rate G_I and the stress intensity factor K_I can be shown (Barsom and Rolfe 1987; Bocca et al. 1991) to be of the form:

$$G_I = \frac{K_I^2}{E'} \tag{3}$$

where for plane stress conditions $E' = E$ and for plane strain conditions $E' = E/(1 - \upsilon^2)$, where υ is Poisson's ratio. Instead of writing the catastrophic crack growth condition in terms of G_I (as $G_I = G_{IC}$), one may alternatively use the condition $K_I = K_{IC}$ (K_{IC} can be obtained using Equation 3 by substituting $G_I = G_{IC}$). K_{IC} and G_{IC} are not loading- and crack-geometry-dependent. Like G_{IC}, K_{IC} also termed the fracture toughness of the material.

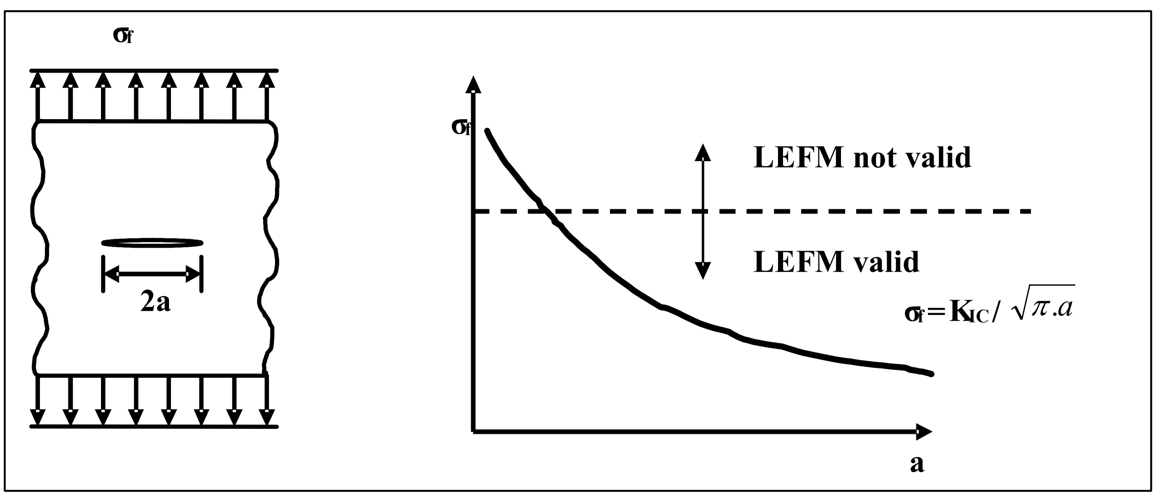

Figure 1. Failure stress related to crack size for infinitely wide plate subjected to tension (Shah et al. 1995).

J-INTEGRAL: The J-integral was presented by Rice (1968) for two-dimensional (2-D) domains containing cracks. Consider a 2-D linear body of linear or nonlinear elastic material free of body forces and subjected to a 2-D deformation field (plane strain, plane stress) so that all stresses σ_{ij} depend only on two Cartesian coordinates (x, y). Suppose the body contains an edge crack as shown in Figure 2.

Define the strain-energy density W as

$$W = W(x, y) = W(\varepsilon) = \int_0^{\varepsilon} \sigma_{ij} \partial \varepsilon_{ij} \tag{4}$$

where $\varepsilon = (\varepsilon_{ij})$ is the infinitesimal strain tensor. Now, define the J-integral as

$$J = \int_{\Gamma} \left(W \partial y - T \frac{\partial u}{\partial x} \partial s \right) \tag{5}$$

As shown in Figure 2, Γ is the curve surrounding the notch tip, the integral being evaluated counterclockwise starting from the lower surface and continuing along the path Γ to the upper surface. In Equation 5, T is the traction vector defined according to the outward normal along Γ, $T_i = \sigma_{ij}\, n_j$: u is the displacement vector, and ds is an element arc length along Γ. Rice (1968a, 1968b) proved the path independent concepts and found that for small-scale yielding the stress energy release rate G is equal to the J-integral. Therefore, the stress intensity factor can be evaluated as follows:

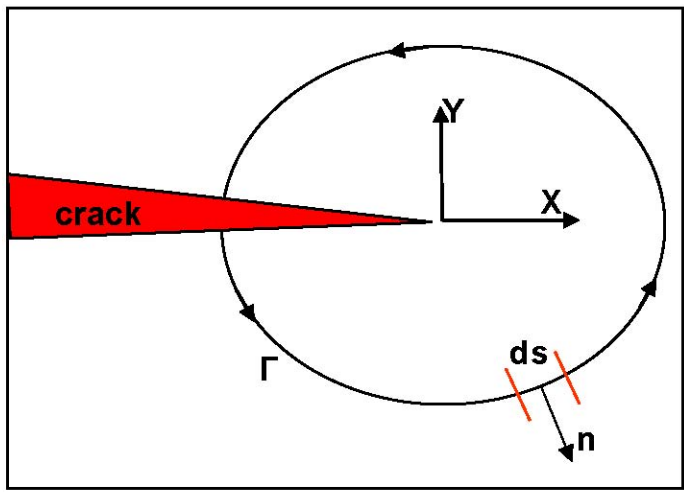

Figure 2. Crack tip coordinate system and typical line integral contour.

$$G = \frac{K_I^2}{E^*} \tag{6}$$

$$J = G = \frac{K_I^2}{E^*} \tag{7}$$

$$K_I = \sqrt{JE^*} \tag{8}$$

where

$E^* = \dfrac{E}{1-\nu^2}$ for plane strain and $E^* = E$ for plane stress

NUMERICAL SOLUTION OF INFINITE PLATE WITH EDGE CRACK: The ABAQUS computer program was used to validate the concept. A semi-infinite plate 40 in. long and 20 in. wide with a 2.0-in. edge crack (Figure 3a) was analyzed. The plate was loaded in pure tension with a uniform pressure of 100 psi (Figure 3b). The numerically calculated J-integral value was 2.93 E-3 psi*in. Assuming a plane stress problem and using Equation 8 the stress intensity factor is calculated as $296.5\,\text{psi}\sqrt{\text{in.}}$ The closed-form solution for this problem (Tada 1973) is $300.0\,\text{psi}\sqrt{\text{in.}}$ The analytical value is 1.16 percent lower than that obtained with the closed-form solution.

Figures 3c and 3d show the displacement and contour plot of the principal maximum stresses around the edge crack. The contour plot (Figure 3d) has a well defined plane stress shape (Anderson 1991). It also shows the stress concentration around the stress raiser, in this case the edge crack.

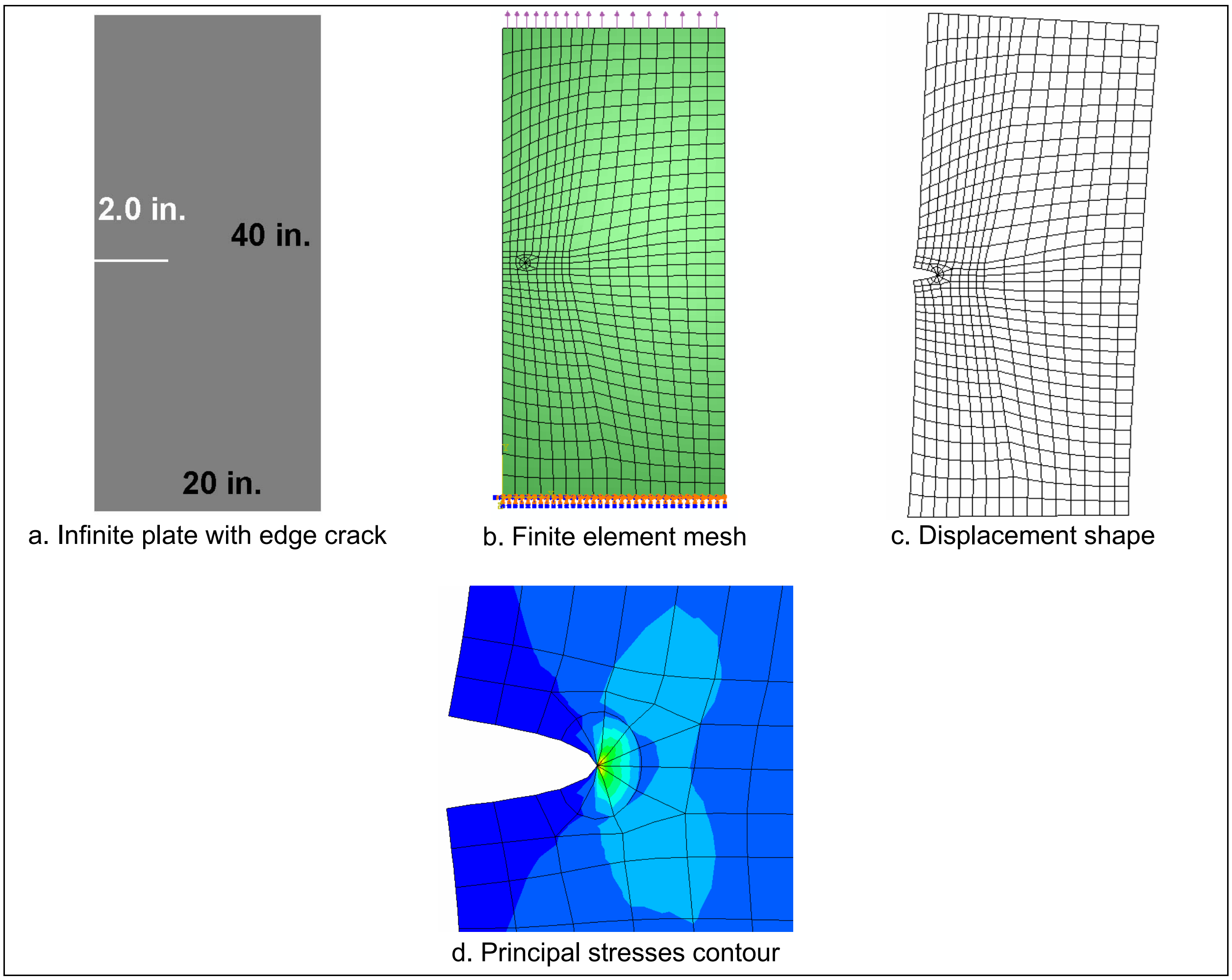

Figure 3. Numerical solution of infinite plate with edge crack.

3-D FINITE ELEMENT ANALYSIS OF MITER GATE WITH MULTIPLE CRACKS: The finite element mesh generation has been one of the most time-consuming issues during a finite element analysis. It is even more complex if a 3-D model is to be analyzed. In the last couple of years new meshing techniques have been developed to easily construct meshes of as-built structures. These techniques allow the modeler to include the actual details of the structures without the need for simplification and assumptions. Figure 4 shows the 3-D mesh of an as-built miter gate. The inset provides a magnification of the bottom right corner of the gate. It becomes clear that advanced meshing techniques are necessary to generate all the details shown in Figure 4.

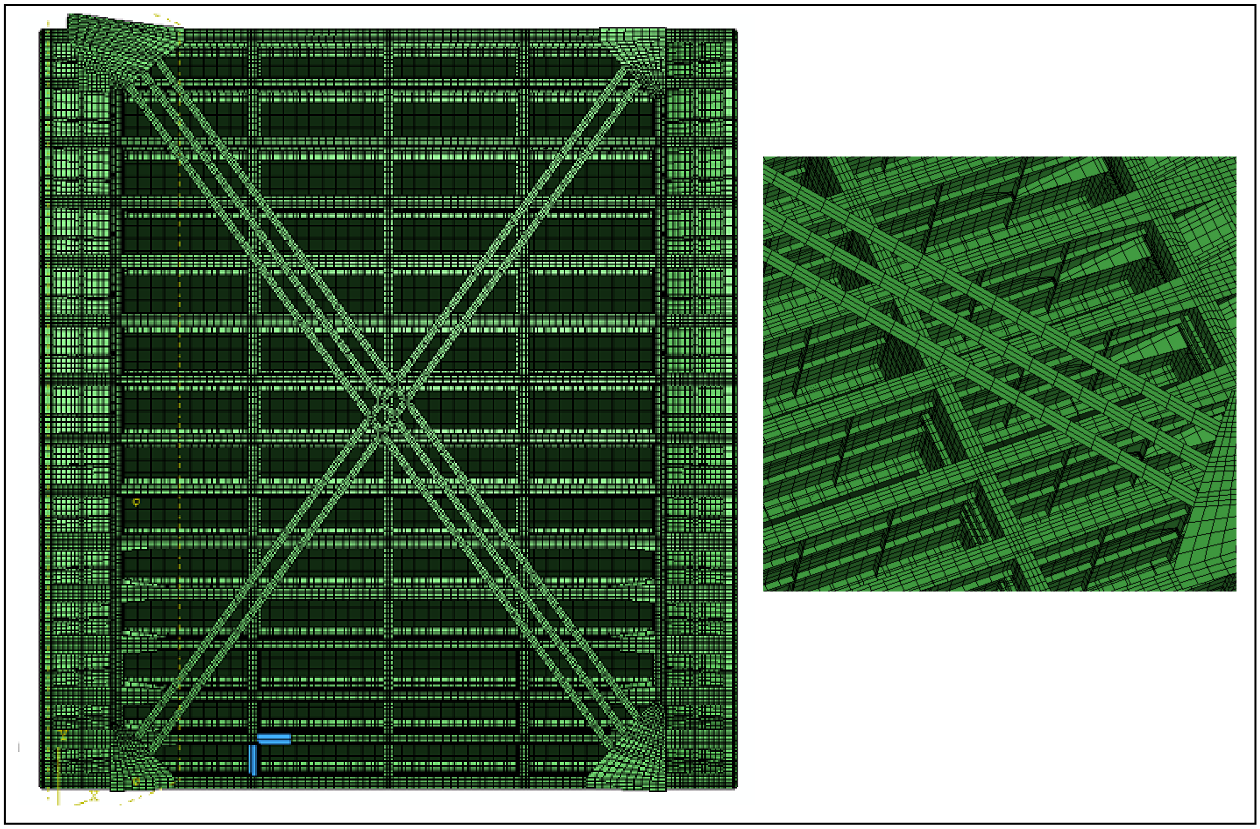

Figure 4. 3-D Finite element mesh of as-built miter gate.

The three steps to generate the more detailed finite element mesh are as follows:

- The 3-D Computer-Aided Design (CAD) drawing of the as-built structure is developed.
- Once the drawing has been generated, an Initial Graphics Exchange Specifications (IGES) file with trimmed surfaces is created. Most of the commercial CAD packages have the capability to generate IGES files.
- The IGES file is imported to the finite element application and the mesh is generated.

With this process all the structural components are easily modeled in the CAD package instead of the finite element mesh application. This utilizes the strengths of both process of software; the

CAD program to create the complex geometry and the finite element program to automatically generate the analytical mesh making the meshing inexpensive.

For this example, the finite element mesh has 364,736 nodes and 123,193 shell elements. Each shell element has a quadratic formulation with five degrees of freedom per node. The total number of degrees of freedom for this model is 1.823 million.

The downstream flange of a vertical diaphragm is shown in Figure 5a and a horizontal girder down stream flange is shown in Figure 5b. These flanges meshes contained artificial diagonal cracks. The meshing required to calculate the J-integral is as follows:

- The crack tip must be meshed with a ring of quarter-point triangular elements.
- All the other elements outside the ring can be quadratic elements.

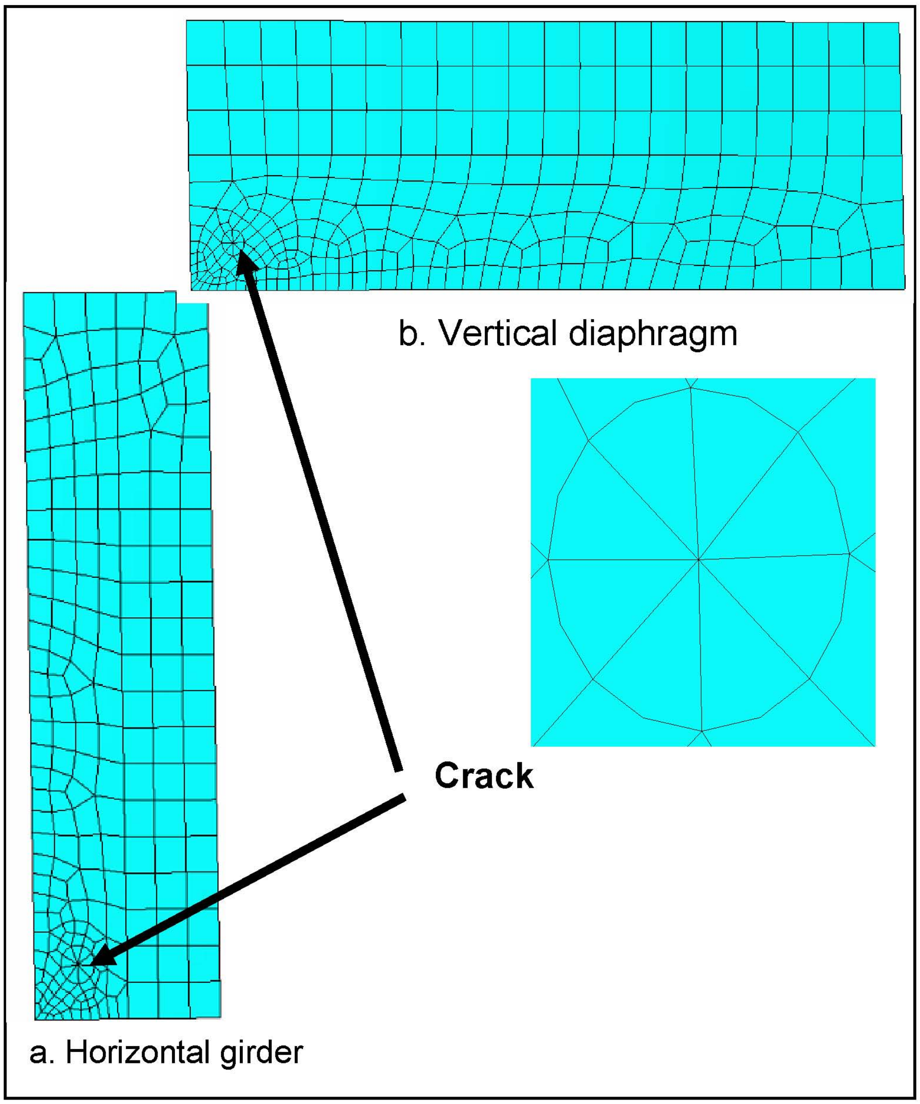

Figure 5. Finite element mesh of crack upstream flange.

For this example, the load boundary condition was a hydrostatic head of 61.5 ft, and the displacement boundary conditions restrained the miter and quoin ends to move along the three principal axes. The movement along the vertical axis at the pintle was also restricted, while the movement along the longitudinal and transverse direction was restrained at the gudgeon.

Figure 6a shows the stress contour of the upstream vertical diaphragm flange. The principal stress at the crack tip is 1,215 psi and the J-integral is 6.91 psi*in. Using Equation 7 and solving for K_I produced a stress intensity factor of $14.397\text{psi}\sqrt{\text{in}.}$, which is much lower than the fracture toughness of the steel (K_{Ic}= $50{,}000\text{psi}\sqrt{\text{in}.}$).

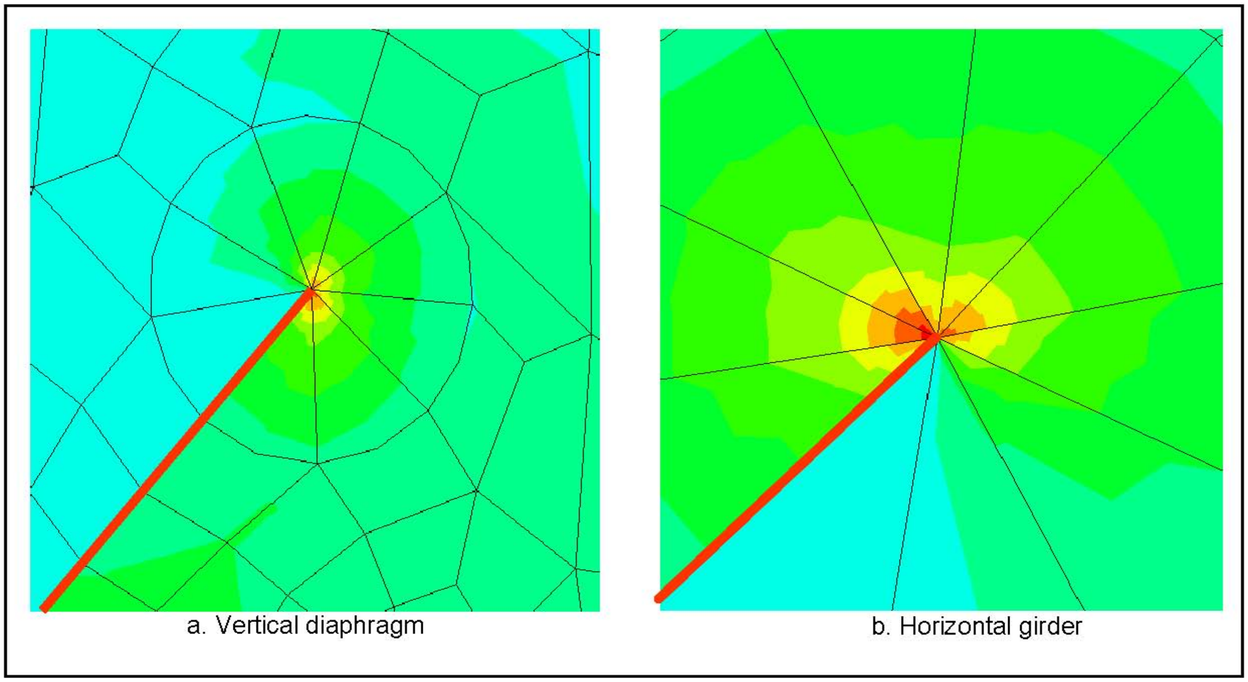

Figure 6. Principal stress contours of the upstream flange.

Figure 6b shows the stress contour of the upstream flange of the horizontal girder. The principal stress at the crack tip is 3,990 psi and the J-integral is 24.66 psi*in. Using Equation 7 and solving for K_I produced a stress intensity factor of $27{,}202\,\text{psi}\sqrt{\text{in}.}$, which is much lower than the fracture toughness of the steel (K_{Ic}= $50{,}000\,\text{psi}\sqrt{\text{in}.}$).

Since the calculated stress intensity factors are less than the fracture toughness of the steel, the cracks on the upstream flanges of the horizontal girder and vertical diaphragm will not propagate. Whether a crack will or will not propagate can, therefore, be predicted accurately. Furthermore, ways of repairing the crack regions numerically can be studied.

CONCLUSIONS: This CHETN has described the capability of calculating stress intensity factors using state-of-the-art analytical techniques. The calculation of the stress intensity factors can be obtained from a 3-D finite element analysis. 3-D finite element meshes can be automatically generated within ABAQUS using IGES-formatted CAD drawings.

The first example showed that the J-integral approach successfully predicted the stress intensity factors for a semi-infinite plate with an edge crack. This example showed a difference of 1.16 percent between the numerically calculated value and the one calculated with text book equations.

In the second example, the 3-D meshing technique utilizing CAD drawings imported into the finite element application was demonstrated. The meshing procedure allows the gates with all its components to be modeled completely without making any simplifying assumptions regarding the geometry. Once again, the stress intensity factors of the upstream flanges of a horizontal girder and a vertical diaphragm were calculated. For this example, the stress intensity factors showed that the cracks will not grow. Therefore repairs can be postponed.

The J-integral methodology can certainly be used to evaluate cracked hydraulic steel structures and to determine remaining life based on the operation conditions. The methodology can also be used to decide the criticality of the cracking problem and whether repairs are needed. Suggested repairs can also be evaluated numerically to predict reliability and safety conditions. These techniques when coupled with a library of all of the Corps hydraulic structures will allow quick analysis whenever problems are found.

ADDITIONAL INFORMATION: For additional information, contact Dr. Guillermo A. Riveros, Information Technology Laboratory, U.S. Army Engineer Research and Development Center, 3909 Halls Ferry Road, Vicksburg, MS 39180 at 601-634-4476, or e-mail: Guillermo.A.Riveros@erdc.usace.army.mil. This effort was funded through the Navigation Systems Research Program. Program Manager is James Clausner, phone (601-634-2009), e-mail James.E.Clausner@erdc.usace.army.mil. This CHETN should be cited as follows:

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